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Proposal for Standard Test of Modulus of Rupture of Concrete with Its Size Dependence

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Recently accumulated test data on the modulus of rupture, as well as analytical studies and numerical simulations, clearly indicate that the flexural strength of concrete, called the modulus of rupture, significantly decreases as the beam size increases. This paper proposes a method to incorporate this size effect into the existing test standards, and focuses particularly on ASTM Standards C 78-94 and C 293-94. The proposed method is based on a recently established size effect formula that describes both the deterministic-energetic size effect caused by stress redistribution within the cross section due to finite size of the boundary layer of cracking at the tensile face of beam, and the classical Weibull-type statistical size effect due to the randomness of the local strength of material. Two alternatives of the test procedure are formulated. In the first alternative, beams of only one size are tested (as is recommended in the current standard), and the size effect on the mean modulus of rupture is approximately predicted on the basis of the average of existing information for all concretes. In the second alternative, beams of two sufficiently different sizes are tested. The latter is more tedious but gives a much better prediction of size effect for the concrete at hand; it allows for the estimation of size effect on not only the mean but also the coefficient of variation of the modulus of rupture (particularly, its decrease with increasing size). Numerical examples demonstrate the feasibility of the proposed approach.

Keywords: concrete; fracture; modulus of rupture; stress.

INTRODUCTION AND RESEARCH BACKGROUND

The flexural strength of concrete beams, known as the modulus of rupture, has long been experimentally studied (Lindner and Sprague 1956; Nielsen 1954; Reagel and Willis 1931; Rocco 1995, 1997; Rokugo et al. 1995; Sabnis and Mirza 1979; Walker and Bloem 1957; Wright 1952; Koide, Akita, and Tomon 1998, 2000), numerically studied (Hillerborg, Modéer, and Petersson 1976; Petersson 1981), and analytically studied (Zhu 1990, Bažant and Li 1995; Bažant and Planas 1998). One result of all this research has been the finding that the modulus of rupture decreases with increasing beam size. This, however, is not yet reflected in the current testing standards.

The cause of the size effect on the modulus of rupture is, for all but the largest-sized specimens, deterministic, stemming from the quasibrittle nature of the material, and particularly the stress redistribution and energy release caused by fracture with a large fracture process zone. For extremely large beam sizes, an additional cause is statistical, stemming from the randomness of local material strength as described by Weibull's (1939) classic theory.

Prior to the 1990s, few structural designers paid any attention to the statistical size effect and none was paid to the deterministic size effect, which is usually much more important. It was commonplace to consider the tensile strength of the material as a constant.

A quarter century ago, however, finite element calculations with the cohesive (or fictitious) crack model by Hillerborg, Modéer, and Petersson (1976) revealed the inevitability of a strong deterministic size effect engendered by stress redistribution within the cross section, due to the strain-softening inelastic response of the material. Petersson (1981) numerically calculated the curve of flexural strength versus the beam depth. He also argued that, for extremely deep beams, an additional statistical Weibull-type size effect that cannot be captured by the deterministic cohesive crack model ought to be taken into account.

As test data accumulated, various empirical formulas were proposed (for example, Rokugo et al. 1995). A simple deterministic formula yielding good agreement with the existing test data was proposed in Bažant and Li (1995) and then refined in Bažant and Li (1996a). This formula was derived on the basis of stress redistribution within the cross section caused by softening in a boundary layer of cracking near the tensile face. The layer was assumed to have a constant (size-independent) thickness, dictated by the size of the inhomogeneities (chiefly the maximum aggregate size). Bažant and Li (1996b) rederived this formula by energy arguments of fracture mechanics, which made it also possible to capture the effect of the geometry of structure and loading in a simple manner, namely, in terms of the derivatives of the energy release function (or stress intensity factor) of an initiating crack with respect to its depth.

On the probabilistic side of the problem, an early study of the stress analysis with the material strength as a random field was published by Shinozuka (1972). Material randomness was simulated by finite elements (for example, Breysse 1990; Breysse and Fokwa 1992; Breysse, Fokwa, and Drahy 1994; Breysse and Renaudin 1996; Roelfstra, Sadouki, and Wittman 1985). Random lattice models exhibiting the quasibrittle size effect were presented by Bažant et al. (1990) and Jirásek and Bažant (1995a, b).

A combination of the statistical and deterministic aspects of the problem has recently been achieved by the probabilistic nonlocal continuum model developed by Bažant and Novák (2000a, b). They showed that this model, unlike the previously developed stochastic finite element models, satisfies the condition, set forth by the classic Weibull theory size effect must ensue as the limit when the ratio of the structure size to the thickness of the boundary layer of cracking (or to the maximum aggregate size) tends to infinity. They also de-

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duced a simple energetic-statistical size effect formula (Bažant and Novák 2000c). This new formula represents asymptotic matching between the deterministic-energetic formula, which is approached for small sizes, and the power-law size effect of the classic Weibull statistical theory, which is approached for large sizes.

The importance of this issue is underscored by recent studies showing that the size effect in flexure of plain concrete beams must have been a significant contributing factor in many disasters, for example, those of the Malpasset Dam (Levy and Salvadori 1992), the Saint Francis Dam (Pattison 1998), and the Schoharie Creek Bridge (Swenson and Ingraffea 1991). The reduction of the effective material strength due to size effect must have been in these structures on the order of 50% (Bažant and Novák 2000c).

The test method according to the current ASTM (1994a, b) Standards C 78-94 and C 293-94 provides the value of the modulus of rupture for one standardized beam size but does not establish the experimental basis for predicting the flexural strength of beams of other sizes. But without such information, the size effect cannot be taken into account during design. Remedying this situation is the goal of this paper.

ENERGETIC-STATISTICAL SIZE EFFECT FORMULA FOR MODULUS OF RUPTURE

The concept of modulus of rupture is based on the elastic beam theory. If the material remains linearly elastic until the maximum load is reached, the strength values obtained by both the flexural test and the direct tensile test will be equal to each other ($f_r = f_t'$).

The modulus of rupture f_r is defined as the maximum normal stress in the beam calculated from the maximum (ultimate) bending moment M_u under the assumption that the beam behaves elastically

$$f_r = \frac{6M_u}{bD^2} \qquad (1)$$

where D, b = beam depth and width. Except for the asymptotic case of an infinitely deep beam, the whole cross section of a concrete beam does not remain elastic up to the maximum load, and so f_r represents merely the nominal strength $f_r = \sigma_N$, which is a parameter of the maximum load having the dimension of strength.

The inelastic behavior before the maximum load is caused by the development of a sizable boundary layer of cracking whose depth is approximately constant, dictated by the maximum aggregate size (Bažant and Planas 1998). The cracking causes energy release and stress redistribution, which increases the moment capacity of the cross section. Because in a deeper beam the cracking layer occupies a smaller percentage of beam depth, there is less stress redistribution, and thus the nominal strength decreases with an increasing beam depth. This represents a size effect.

The size effect on the modulus of rupture has been shown to follow the energetic-statistical formula (Bažant and Novák 2000c)

$$f_r = f_r^0\left[\left(\frac{D_b}{D}\right)^{rn/m} + \frac{rD_b}{D}\right]^{1/r} \qquad (2)$$

where f_r^0, D_b, r, and m are positive constants representing unknown empirical parameters; and n is the number of dimensions in geometric similarity—n = 2 or 3 (D_b has approximately the meaning of a boundary layer of cracking). Because r and m can be prescribed on the basis of the information on all concretes studied in the literature, there are only two parameters, namely f_r^0 and D_b, to be identified from tests of the given concrete. For this purpose, testing beams of only one size while ignoring the size effect, as currently specified in standards, is insufficient. One must either test beams of two sufficiently different sizes, or make a size effect correction based on prior knowledge.

Data fitting with the new formula (2) reveals that, for concrete and mortar, the Weibull modulus $m \approx 24$ rather than 12, the value currently accepted (Bažant and Novák 2000c). This means that, for extreme sizes, the nominal strength (modulus of rupture) decreases for two-dimensional (2D) similarity (n = 2), as the –1/12 power of the structure size, and for three-dimensional (3D) similarity, as the –1/8 power (in contrast to the –1/6 and –1/4 powers that have generally been assumed so far). Fitting by this formula to the main test data sets available in the literature showed an excellent agreement, with a rather small coefficient of variation of errors of the formula compared to the test data. Furthermore, the new formula was verified by numerical simulations with the nonlocal Weibull theory (Bažant and Novák 2000a, b).

PROPOSAL FOR SIZE EFFECT EXTENSION OF CURRENT STANDARD TEST

The entire procedure of the standard test method can be retained. Only the size effect consideration needs to be added. Two levels of size effect consideration are proposed: 1) testing with only one specimen size and taking the size effect into account based on prior knowledge, and 2) testing with two specimen sizes. The latter is more accurate but involves more work. For both levels, the values

$$m = 24, \quad r = 1.14, \quad n = 2 \qquad (3)$$

which have been shown to be suitable for all concretes on the average (Bažant and Novák 2000c), should be used.

Testing with only one specimen size and crude estimate of l_0

1. When the ease of testing is important, one specimen size suffices; at least D_1 = 76 mm (3 in.) should be used, but it is better to use D_1 = 305 mm (12 in.). The uncertainty of the test results depends on the size selected, as numerically verified by nonlocal Weibull theory (Bažant and Novák 2000a, b). The scatter is much higher for smaller sizes; for example, the coefficient of variation of deviation of the formula from test data, $\omega \approx 0.3$ for D_1 = 76 mm, while $\omega \approx 0.1$ for D_1 = 305 mm). Therefore, more specimens are desirable if the smaller size

is used, but generally it is recommended that the number of specimens of one size should be no less than six.

2. Using the existing formula in the ASTM standards C 78-94 and C 293-94, the modulus of rupture can be determined as the mean value f_1 (in MPa) corresponding to the selected size D_1.

3. The parameter D_b of the size effect formula (2) is then approximately estimated as a function of the characteristic length l_0

$$D_b \approx \delta_1 10^{0.15 + (l_0/l_1)}, \quad \delta_1 = 1 \text{ mm}, \quad l_1 = 53 \text{ mm} \qquad (4)$$

(Justification of this formula will be given later.) The characteristic length l_0 is usually not known, and a rough estimate may then be obtained as

$$l_0 \approx d_a(d_a/\delta_1)^{1/3}, \quad \delta_1 = 1 \text{ mm} \qquad (5)$$

where d_a is the maximum aggregate size. (It is convenient, albeit not required, to give aggregate size in mm.)

4. Knowing D_b, one can estimate

$$f_r^0 = f_1\left[\left(\frac{D_b}{D_1}\right)^{rn/m} + \frac{rD_b}{D_1}\right]^{-1/r} \qquad (6)$$

All the parameters of the energetic-statistical formula (2) for size-dependent prediction of modulus of rupture are thus determined. For any size D, modulus of rupture f_r can be easily calculated.

Testing with two specimen sizes

1. When more accurate results are desired, two specimen sizes need to be used, for example

$$D_1 = 76 \text{ mm (3 in.) and } D_2 = 305 \text{ mm (12 in.)} \qquad (7)$$

Two other sizes can also be selected, but note that the sizes selected must not be very close (such as $D_1 = 76$ mm and $D_2 = 100$ mm). If the sizes are not very different, the problem of identification of material constants tends to be ill-posed, and the experimental scatter tends to cause significant uncertainty (Bažant and Li 1996b; Planas, Guinea, and Elices 1995; Bažant and Planas 1998). The number of specimens should be chosen as previously discussed.

2. According to the existing formula in ASTM standards C 78-94 and C 293-94, the values of the modulus of rupture are calculated for each individual size: f_1 for size D_1, and f_2 for size D_2.

3. The unknown parameters f_r^0 and D_b of the size effect formula (2) are then solved from the following system of two nonlinear equations that follows from writing the formula (2) for $D = D_1$ and $D = D_2$, and solving f_r^0 from each

$$f_r^0 = f_1\left[\left(\frac{D_b}{D_1}\right)^{rn/m} + \frac{rD_b}{D_1}\right]^{-1/r} \qquad (8)$$

$$f_r^0 = f_2\left[\left(\frac{D_b}{D_2}\right)^{rn/m} + \frac{rD_b}{D_2}\right]^{-1/r} \qquad (9)$$

Equating the last two expressions yields a formula for D_b

$$D_b = \left(\frac{f_1^r D_1 D_2^p - f_2^r D_1^p D_2}{r(f_2^r D_2 - f_1^r D_1)}\right)^{1/p}, \quad p = \frac{rn}{m} \qquad (10)$$

4. Parameter f_r^0 is then evaluated from Eq. (8) or (9). The energetic-statistical formula (2) for size-dependent prediction of the modulus of rupture is thus completely determined. For any size D, the modulus of rupture f_r can be easily calculated.

JUSTIFICATION OF D_b ESTIMATE FOR ONE-SIZE TESTING

For one-size testing, an estimate of unknown parameter D_b is needed. For this purpose, all the well-documented relevant test data available in the literature, consisting of 10 data sets from eight different laboratories, were analyzed (Lindner and Sprague 1956; Nielsen 1954; Reagel and Willis 1931; Rocco 1995, 1997; Rokugo et al. 1995; Sabnis and Mirza 1979; Walker and Bloem 1957; Wright 1952). The values of the parameters of the energetic-statistical formula (2) for each individual data set were obtained by fitting the test data. The results and all data points are plotted in Bažant and Novák (2000c). The aim was to obtain a prediction formula for D_b as a function of some simple characteristics of concrete—at least a rough approximate prediction, based, for example, on the maximum aggregate size. Although a very good prediction seems impossible, the following procedure has led to useful results.

The boundary layer thickness D_b may be assumed to be affected by the basic fracture characteristics of concrete, such as the fracture toughness, the fracture energy, the effective length of fracture process zone, or the characteristic length. Therefore, the size effect method (Bažant and Planas 1998) has been utilized to determine these characteristics for each individual data set considered, exploiting the relation

$$f_r = \frac{K_{Ic}}{\sqrt{D}k(\alpha_0 + cf/D)} \qquad (11)$$

Here, K_{Ic} is the fracture toughness and c_f is the fracture process zone length, whose values for each data set have been obtained by nonlinear fitting of the size effect data (plot of f_r versus D) for that set using, for example, the Levenberg-Marquardt algorithm: $k(\alpha_0 + c_f/D)$ is the dimensionless stress intensity factor (depending on the structure geometry) as a function of relative crack length $\alpha = \alpha_0 + c_f/D$ at the start of crack propagation (which triggers failure); and α_0 is the relative notch length, which is zero in this case of unnotched specimens. For three-point and four-point bending, the values of $k(\alpha)$ can he obtained, for example, from Tada, Paris, and Irwin's (1985) handbook.

Once K_{Ic} and c_f for each data set had been obtained by data fitting with Eq. (11), then the characteristic length l_0 was calculated for each set as

$$l_0 = (K_{Ic}/f_t')^2 \qquad (12)$$

where f_t' is the direct tensile strength of concrete. The direct tensile strength values were not reported and thus were estimated from the reported splitting tensile strength or the compression strength. The plot of log D_b versus l_0 for all the data

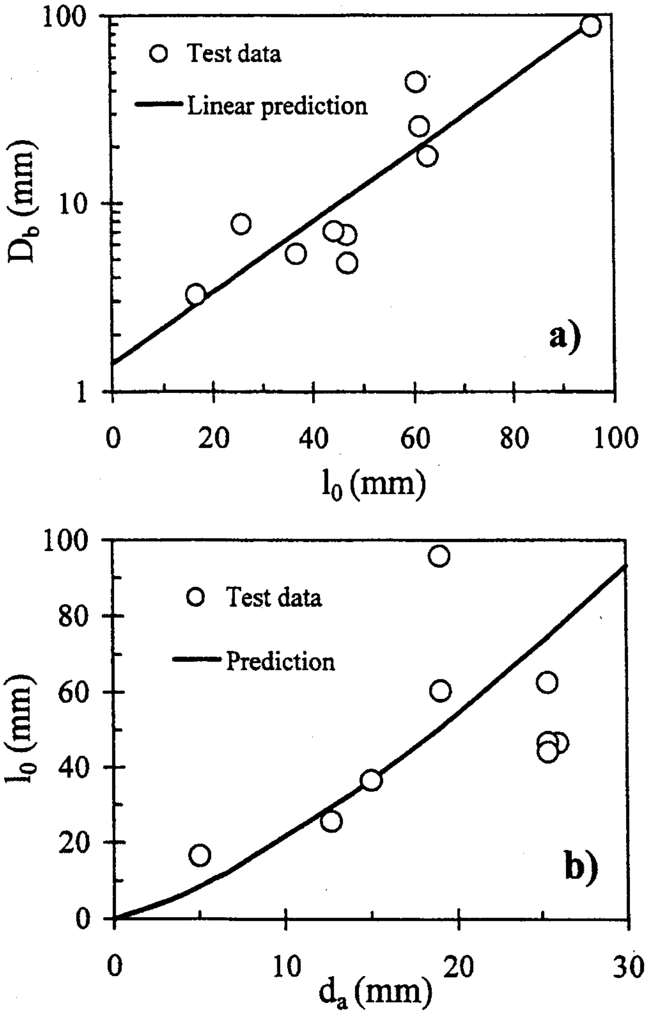

Fig. 1—(a) Dependence of log D_b *on material characteristic length* l_0 *determined for various data sets; and (b) dependence of* l_0 *on maximum aggregate size* d_a.

sets is shown in Fig. 1(a). In spite of a large scatter, one can discern a trend. The trend may be expressed in the form of Eq. (4), shown as the solid line in Fig. 1(a). Plotting the characteristic length versus the maximum aggregate size for each data set results in Fig. 1(b), where the trend of the data, shown as the solid line, is expressed by Eq. (5).

PROBABILISTIC PREDICTION

Statistically, what the energetic-statistical formula (2) predicts is the mean size effect curve, because it has been developed for the means of modulus of rupture. Formulas in the form of Eq. (2) could also be used for the medians, as they usually differ only slightly from the means. But for describing the size effect on low or high percentiles of modulus of rupture, formulas of such a form would be incorrect.

For small sizes, the scatter of the modulus of rupture is generally larger than for large sizes. This fact is evidenced by the existing test data and has also been verified by numerical simulation with the nonlocal Weibull theory (Bažant and Novák 2000a, b). If the energetic-statistical formula were used to fit, for example, the 5 and 95 percentiles for each data set, the resulting curves could even intersect for large sizes, which would be conceptually wrong. Therefore, a different approach is necessary to predict the statistical scatter.

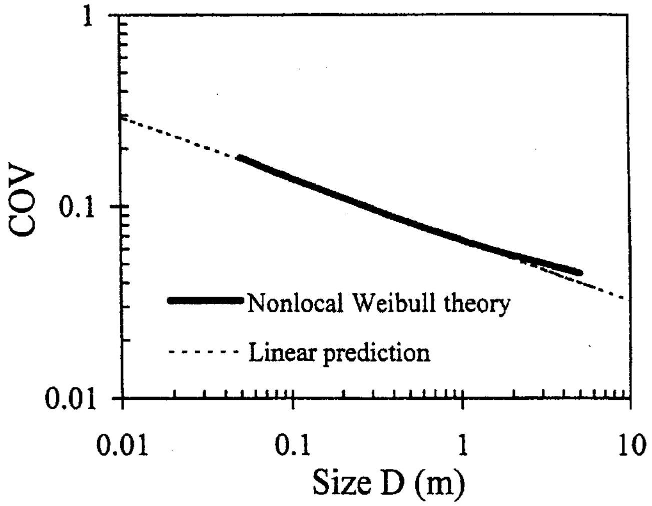

Fig. 2—Dependence of coefficient of variation ω *of* f_r *on beam size (depth)* D*, obtained by numerical simulation with nonlocal Weibull theory.*

Information obtained from one-size testing does not suffice to predict the decrease of scatter with the size of a specimen. Therefore, testing with at least two (significantly different) specimen sizes is necessary for predicting the scatter of the modulus of rupture.

Statistical information on the scatter can be obtained from the two data sets corresponding to the two sizes selected. The coefficients of variation ω of the modulus of rupture are calculated for each individual size in the standard statistical way (the standard deviation divided by the mean): ω_1 for size D_1, and ω_2 for size D_2. They characterize the variability of test results in a relative manner.

Normally, $\omega_1 \geq \omega_2$. If not, it is likely that the tests were not performed properly—human errors, an insufficient number of specimens, specimen sizes not sufficiently different, inadequate test control, and poor measuring devices could all be factors. The case $\omega_1 < \omega_2$ can occur for statistical reasons. Its treatment would require linking the quality of estimate and the number of tests, which is beyond the scope of this paper and destroys the simplicity of scatter prediction. Therefore, it is recommended that in such cases the probabilistic prediction be skipped. A scatter increasing with the specimen size is simply not realistic.

The aim is to predict ω for any size as information additional to the mean size effect curve described by the energetic-statistical formula (2). The general trend of ω versus size D needs to be identified for this purpose.

Numerical simulations with the nonlocal Weibull theory (Bažant and Novák 2000a, b) revealed an almost linear relationship between $\log \omega$ and $\log D$, as shown by the solid line in Fig. 2. Deviations in the logarithmic scale can be observed only for very large sizes, but the scatter for these sizes is generally so small that the error of a linear relationship between $\log \omega$ and $\log D$ can be neglected. The linear dependence of $\log \omega$ on $\log D$ may be written as

$$\log\omega = \frac{\log\omega_2 - \log\omega_1}{\log D_2 - \log D_1}\log D + \frac{\log\omega_1 \log D_2 - \log\omega_2 \log D_1}{\log D_2 - \log D_1} \quad (13)$$

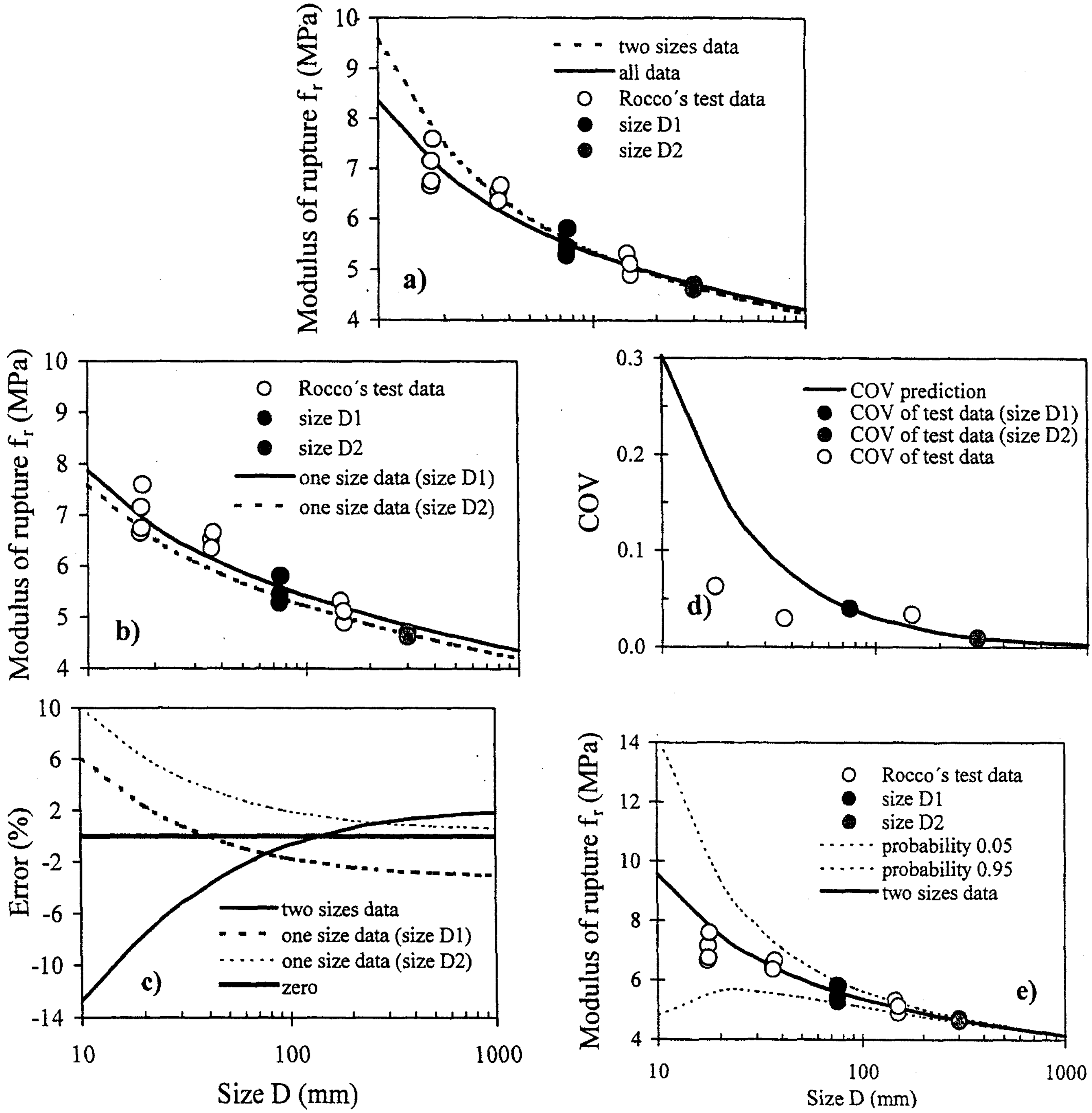

Fig. 3—Modulus of rupture prediction using Rocco's data: (a) dependence of f_r *on beam size (depth)* D, *according to energetic-statistical formula (2) obtained on basis of either all Rocco's data or his data for two sizes only; (b)* f_r *obtained on basis of one-size testing only; (c) error of using test data for two sizes and one-size data only; (d) estimation of coefficient of variation* ω*; and (e) probability bounds (5 and 95 percentiles).*

Then, considering the normal probability distribution for modulus of rupture to be acceptable, one can easily estimate any percentiles of probability cut-off. For example, the 5 and 95 percentiles are calculated for normal distribution as (meanf_r)($1 \pm 1.645\omega$). Despite the heuristic basis of this prediction, reasonable results are achieved in comparison with tests, as shown in the following section.

NUMERICAL EXAMPLES

To demonstrate the procedure, two sets of data will be considered. First, consider Rocco's (1995, 1997) data, which represent excellent data on the modulus of rupture, with five different sizes spanning the broadest size range among all the available data. Suppose that only two specimen sizes were used in these tests, $D_1 = 75.28$ mm ($\approx$ 3 in.) and $D_2 = 304.8$ mm (12 in.). The mean values of the modulus of rupture for these individual sizes are: $f_1 = 5.60$ MPa for size D_1, and $f_2 = 4.67$ MPa for size D_2. The number of specimens tested for each size is only four in this case. According to the foregoing procedure, two unknown parameters of the energetic-statistical formula (2) are determined: $f_r^0 = 6.36$ MPa and $D_b = 5.55$ mm.

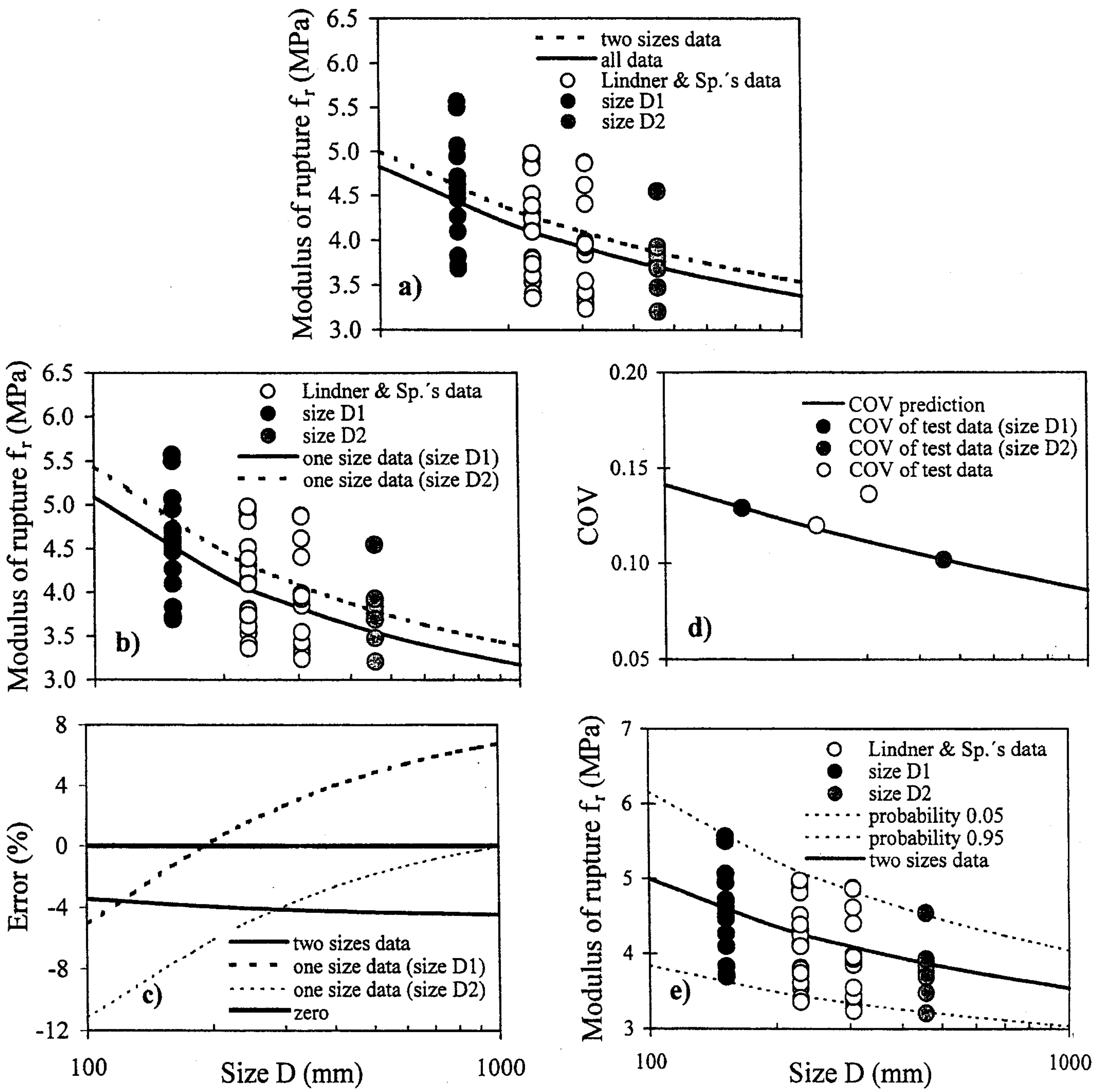

Fig. 4—Modulus of rupture prediction using Lidner and Sprague's data: (a) dependence of f_r *on beam size (depth)* D*, according to energetic-statistical formula (2) obtained on basis of either all data or data for two sizes only; (b)* f_r *obtained on basis of one-size testing only; (c) error of using test data for two sizes and one-size data only; (d) estimation of coefficient of variation* ω*; and (e) probability bounds (5 and 95 percentiles).*

Taking all five sizes of Rocco's data into account, Bažant and Novák (2000c) obtained, by nonlinear fitting of the energetic-statistical formula, the values $f_r^0 = 6.78$ MPa and $D_b = 3.25$ mm as the best possible estimates. From the comparison shown in Fig. 3(a), one can now see that the differences between the result based on all the sizes and the result based on only two sizes are negligible. Figure 3(b) shows the hypothetical results of one-size testing considering either of the two sizes. Based on maximum aggregate size $d_a = 5$ mm, the characteristic lengths $l_0 = 8.55$ mm and $D_b = 2.05$ mm were estimated according to the formulas, Eq. (4) and (5). The error may be defined as the difference between the value of the modulus of rupture measured for each size and the value predicted on the basis of the data either for the two chosen sizes or for the chosen single size, and expressed as a percentage of the mean. A plot of this percentage error versus the size, shown in Fig. 3(c), shows a good enough agreement. For normal cross-section dimensions used in the concrete industry, the error is in this case below 4% of the mean. The prediction of the coefficient of variation ω versus size D according to the formula (13) is plotted in Fig. 3(d), for which $\omega_1 = 0.04$ and $\omega_2 = 0.01$. The 5 and 95 percentile curves (that is, the limits having probabilities exceeding 0.05 and 0.95) are shown in Fig. 3(e).

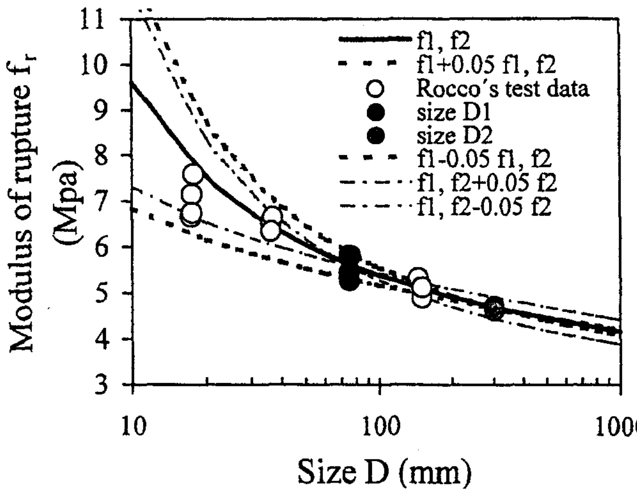

Fig. 5—Sensitivity analysis—dependence of predicted modulus of rupture on level of accuracy of means f_1 and f_2.

Second, consider Lindner and Sprague's (1956) data, for which the scatter of test results is much larger; see Fig. 4(a). Suppose that only two specimen sizes are used in the analysis of these tests, particularly the sizes D_1 = 152.4 mm (6 in.) and D_2 = 457.2 mm (18 in.), for which the values of the means are f_1 = 4.48 MPa and f_2 = 3.79 MPa. The number of specimens was 17 for the first size and eight for the second size. Then one obtains f_r^0 = 4.77 MPa and D_b = 16.3 mm (while the fitting of all the data sets combined yields f_r^0 = 4.61 MPa and D_b = 17.89 mm).

The one-size testing is again considered for each size (Fig. 4(b)). The characteristic length l_0 = 74.66 mm and the thickness D_b = 36.20 mm were estimated on the basis of the maximum aggregate size, d_a = 25.4 mm. The errors for the one-size testing are, in this particular case, much better balanced than those obtained for the single-size testing. The plot also indicates that the larger the size used in the one-size testing, the more reliable the results. Figure 4(d) shows the plot of ω versus size D, and Fig. 4(e) shows the 5 and 95 percentile curves (ω_1 = 0.13, ω_2 = 0.10).

To illustrate the sensitivity of prediction of the modulus of rupture on the level of accuracy of the means f_1 and f_2, Fig. 5 shows how an error in the estimation of the means, such as ±5% perturbations, influences the size effect curve in the case of Rocco's data. This small error can influence the estimates of D_b and f_r^0 significantly (the minimum and maximum values for the four possible combinations of the perturbed values are D_b = 1.04 and 11.4 mm, and f_r^0 = 5.56 and 7.83 MPa). But the overall prediction of the size effect on the modulus of rupture is not overly affected, especially for larger sizes that are used in the concrete industry.

The examples show that, with a proper testing procedure, two sufficiently different specimen sizes suffice to achieve the same test result as does using many specimen sizes. Thus it transpires that two-size testing can adequately characterize the statistical variability of modulus of rupture, particularly the decrease of scatter with size.

SPREADSHEET FORM FOR PROPOSED STANDARD TEST

To make the determination of the energetic-statistical size effect formula parameters particularly easy, a spreadsheet form, available from the authors, has been developed for a common computer software program. One needs to open the computer file, "FSCtest.xls" and type input parameters (highlighted in yellow). The output parameters (parameters of the size effect formula) are calculated automatically (highlighted in red). The illustrative size effect figures are plotted automatically when the input parameters are changed, with the important points being highlighted.

SUMMARY AND CONCLUSIONS

1. It is proposed that the existing ASTM standards C 78-94 and C 293-94 for the modulus of rupture test be extended by testing for the size effect. The proposed method includes both the deterministic (energetic) and statistical size effects. Two alternatives of the test procedure are formulated.

2. In the first alternative, the size effect on the mean modulus of rupture is approximated on the basis of the existing information for all concretes on the average.

3. In the second alternative, beams of two sufficiently different sizes are tested. The latter is more tedious but gives a much better prediction of size effect for the concrete at hand and allows estimating not only the size effect on the mean but also the size effect on the coefficient of variation of the modulus of rupture, characterized by a decrease of the coefficient of variation with increasing size.

4. Numerical examples demonstrate feasibility of the proposed approach.

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APPENDIX I—PROPOSED CHANGES IN TEXT OF ASTM STANDARDS C 78-94 AND C 293-94

The proposed standard test method can be specified as follows:

1. **Scope** (Unchanged)

2. **Referenced Documents** (Unchanged)

3. **Significance and Use**

3.1. (Proposed to add to the end of paragraph) Because of clear experimental evidence and relevant theory that the flexural strength of concrete, called modulus of rupture, significantly decreases as the beam size increases, a feasible method to determine this size dependence (size effect) is implemented. It enables the use of size-dependent value of modulus of rupture in the design process to ensure desired reliability level of design.

3.2. (Unchanged)

4. **Apparatus** (Unchanged)

5. **Testing**—C 78-94, **Test Specimen**—C 293-94

5.1. (Proposed to add) One of two levels of standard testing may be chosen: testing with only one specimen size (level one) and testing with two specimen sizes (level two). Level one is easier; level two is more accurate. Level one: use beam specimens of depth D_1 = 76 mm (3 in.). Level two: use specimens of two sizes. D_1 = 76 mm (3 in.) and D_2 = 305 mm (12 in.) with the same cross-section width and proportionally increased spans (so as to maintain geometric similarity. The number of specimens for each size should be no less than six. However, for the smaller, size higher number of specimens (for example, 12) it is strongly recommended to achieve a better prediction.

5.2. (Unchanged)

6. **Procedure**

6.1. to 6.2. (Unchanged)

6.3. (Proposed to add) To minimize the rate effect, the time to reach the maximum load should be approximately the same for different sizes. This is achieved roughly when the beam deflection rate at the load point is proportional to the beam depth.*

7. **Measurement of Specimens After Test** (Unchanged)

8. **Calculations**

8.1 to 8.3. (Unchanged)

8.4. –C 78-94, 8.2. –C 293-94 (Proposed) To identify the coefficients of the energetic-statistical size effect formula, the following parameters may be assumed for normal concretes: r = 1.14, m = 24, and n = 2.

Level One—

The modulus of rupture for specimen size D_1 is determined according to the formula in Section 7.1., and is denot-

*This would be exactly true if the flexural strength were size independent. More precisely, the deflection rate that has been increased in proportion to the size should further be decreased in proportion to the expected size effect. But this correction is difficult to implement because the size effect is not yet known while being at the same time unimportant because only order of magnitude changes of the applied deflection rate have any significant effect on the results. The reason that the time to maximum load should be the same is that this ensures the strain rate at homologous points of the beams of different sizes are the same, and in particular, the boundary layer of cracking (or the fracture process zone) in beams of different sizes are strained at the same rate.

ed as f_1. The parameter D_b of the size effect formula is then approximately estimated using Eq. (4) as a function of characteristic length of concrete, which, if unknown, may in turn be estimated from the maximum aggregate size. Then f_r^0 is calculated from Eq. (9).

Level Two—

According to the formula in Section 7.1., the modulus of rupture values are first determined as the mean values for each individual size: f_1 for size D_1 and f_2 for size D_2. The unknown parameters f_r^0 and D_b of the size effect formula (2) are then solved using Eq. (8) to (10). The size-dependent formula (2) for the modulus of rupture is thus completely determined. From this formula, the modulus of rupture f_r may then be easily calculated for any size D. Estimation of the expected scatter in the form of coefficient of variation ω can be done using the formula (13).

9. **Report**

9.1.1 to 9.1.10. (Unchanged)

9.1.11. (Proposed) Size effect formula with all parameters used and identified by procedure stated above: additionally, a plot of modulus of rupture versus size would be generally illustrative and helpful.

10. **Precision and Bias** (Unchanged)

11. **Keywords**

11.1. (Proposed to add) size effect