

**Distributions of Ground Movements Parallel to Deep Excavations in
Clay**

Jill Roboski¹

Research Assistant, Department of Civil and Environmental Engineering, Northwestern
University, Evanston, IL, 60208

and

Richard J. Finno

Professor, Department of Civil and Environmental Engineering, Northwestern
University, Evanston, IL, 60208, r-finno@northwestern.edu

¹ Currently, Senior Staff Professional, GeoSyntec Consultants, Chicago, IL, 60603,
JRoboski@GeoSyntec.com

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ABSTRACT

An empirical procedure for fitting a complementary error function (erfc) to settlement and lateral ground movement data in a direction parallel to an excavation support wall is proposed based on extensive optical survey data obtained around a 12.8 m excavation in Chicago. The maximum ground movement and the height and length of an excavation wall define the erfc fitting function. The erfc fit is shown to apply to three other excavation projects where substantial ground movement data were reported.

KEYWORDS

excavations, clays, ground movements, performance data

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INTRODUCTION

Evaluating the magnitude and distribution of ground movements adjacent to an excavated wall is an important part of the design process when excavating in an urban environment. In many cases, three-dimensional effects caused by the relatively high stiffness at the corners of an excavation lead to smaller ground movements near the corners and larger ground movements towards the middle of the excavation wall. Variations in settlement develop parallel to the wall as a result of this difference in magnitude, yet there are no published methods that allow a designer to estimate these settlement distributions.

It is well known that buildings adjacent to excavations can be damaged by excavation-induced distortions. For example, Boone et al. (1999), Cording et al. (1999), Burland (1995), Boscardin and Cording (1989), Boscardin (1980), O'Rourke et al. (1977), and Burland and Hancock (1977) all described cases where damage occurred in structures from distortions arising from adjacent excavations. In these studies, the focus was on distortions that arise from the distribution of movements normal to an excavation wall. Finno and Bryson (2002) described a case study in which a school supported by shallow foundations paralleled a deep excavation in Chicago. The footings for the school were within 1.6 m of the excavation support system. Damage due to distortions that arose from the distribution of movements in a direction parallel to the excavation was observed on the exterior of the building as hairline cracks in the foundation wall and the

exterior masonry wall. These cracks were observed when distortions reached 1/1,500 in the direction parallel to the excavation. This study emphasized the fact that distortions that arise in a direction parallel to an excavation also can damage a structure.

A number of semi-empirical methods can be used during the design phase of a supported excavation project to estimate the magnitude of the ground movements (e.g., Peck 1969; Clough et al. 1989; Clough and O'Rourke 1990) and their distribution perpendicular to the excavation (e.g., Clough and O'Rourke 1990; Hsieh and Ou 1998). For some projects, these estimates are supplemented by results of finite element analyses. However, there is no guidance in literature for estimating the distributions of the induced movements in a direction parallel to an excavated wall.

Recently, optical survey data collected in conjunction with the excavation for the Robert H. Lurie Medical Center in Chicago (Finno and Roboski 2005) were detailed enough to develop an empirical relation that correlates the distribution of movements in a direction parallel to an excavation support wall to the height and length of an excavation, provided that the maximum settlement and/or lateral ground surface movement is known or can be estimated. This paper presents a complementary error function that was developed to describe the distribution of ground movements parallel to an excavation wall, summarizes the procedures used to find the required coefficients, and illustrates how the function fits data at the Lurie Center excavation site. Reasonable estimates of the distortions parallel to the Lurie Center excavation site also are found from the complementary error function fit. The procedure is subsequently applied to three additional excavation case studies where sufficient ground surface movement data were collected to allow a parallel distribution to be defined. The fitting procedure is shown to

apply to the movements that developed at these excavations made through differing soil conditions, with varying support systems.

DEFINITION OF COMPLIMENTARY ERROR FUNCTION

Profile functions, or fitting functions, have been used by various investigators to model free field ground movements due to mining subsidence, tunneling and excavation (Ahmed 1990). One common profile function is the normal probability, or normalized Gaussian, distribution. Several researchers have employed this function to describe the profile of a settlement trough over a tunnel (Peck 1969; O'Reilly and New 1982; Attewell et al. 1986; Mair et al. 1993). In some instances, this normal probability function may have been erroneously named an error function. In fact, the error function (erf) is the integral of the normal probability distribution. It is defined as:

$$[1] \quad erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

where $erf(0) = 0$ and $erf(\infty) = 1$.

The complementary error function ($erfc$) is defined as:

$$[2] \quad erfc(x) = 1 - erf(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$$

The complementary error function, shown as a solid line in Figure 1, forms the basis of the function that is fitted herein to the extensive ground movement data collected at the excavation for the Lurie Research Center in Chicago (Finno and Roboski 2005). The form of this fitting function is:

$$[3] \quad y = C * erf \left[\frac{(x - A)}{B} \right] + D$$

where x is the distance from the origin, A is the distance to the inflection point of the complementary error function from the origin, B is the shape parameter which controls the spread or slope at the inflection point, C is the parameter which sets the amplitude, and D is the parameter which controls the vertical offset.

As illustrated in Figure 1, the fitting parameters shift or stretch the shape of the function and change the position of the inflection point. The solid line, $y(x)=erfc(x)$ is the basic fitting function. In this case, $A=0$, $B=1$, $C=1$, and $D=0$. By definition, the amplitude of the basic complementary error function is equal to 2. The fitting parameter C is used to vary this amplitude such that the amplitude is expressed as $2C$. The slope of a line drawn tangent to the inflection point controls the spread of the erfc curve. This slope is illustrated in Figure 1, and is inversely proportional to the parameter B . The dash-dot line, $y(x)=erfc(x/2)$, is identical to the first line ($A=0$, $C=1$, and $D=0$) except the shape of the curve has been spread out such that $B=2$. The dash line in Figure 1, $y(x)=-2+erfc(x-2)$; has been shifted in both the x - and y -directions, from the original position of the solid line. In this case the parameters are $A=2$, $B=1$, $C=1$, $D=-2$. The function is shifted down the y -axis two units because $D=-2$, and the inflection point is shifted two units along the x -axis in the positive direction because $A=2$.

Figure 2 is a schematic which describes the erfc parameters used to fit ground movement data. While other fitting functions are possible, the shape of the erfc function is appropriate for fitting movement data because it exhibits a gradually sloping profile from small movements at the corners of an excavation to a maximum movement near the center of an excavation, an inflection point and a plateau-like representation of the near

maximum movements near the center of a wall. Prior to fitting, the ground movement data obtained at a constant distance perpendicular to the wall is plotted versus distance to the closest corner of the excavation. For this reason, x is less than or equal to $L/2$. This data can be either settlements or lateral movements. The value of D is the maximum-recorded ground movement measured at some distance behind the excavation, δ_{\max} , and has the same sign (positive or negative) as the ground movement data being fitted. Assuming the distribution of movements parallel to the excavation is bounded by zero and δ_{\max} , the amplitude, $2C$, is equal to D , the maximum ground movement. The sign of C must be opposite of the sign of D to obtain the proper form of the erfc. Defining C and D in this manner allows the erfc fitting function to be determined with two parameters related to the geometry (A and B) and a maximum value of ground movement. The fitting function is calculated for half the excavation wall, and therefore must be mirrored about the centerline to obtain a distribution along the wall. It is possible for the distribution to extend beyond the corners of the excavation (i.e. ground movement magnitude may not be zero at the excavation corners).

Following the approach of Schmidt (1969), the distance from the corner of the excavation to the inflection point, A , is determined by plotting the ground movements versus the square of the distance from the centerline on a semi-log plot as shown in Figure 3. A best-fit line drawn through the data intersects the vertical axis at the maximum movement value. The magnitude of the ground movement at the inflection point is one-half the maximum ground movement value; this value is indicated on Figure 3 by a dashed line at a magnitude of 13 mm. The square root of the intersection of this

value and the best-fit line occurs at the distance from the centerline to the inflection point, i . The value of A is one-half the excavation length minus the value of i .

The slope of the tangent at the inflection point is inversely proportional to the value B as shown in Figure 2. This slope controls the spread of the erfc. Assuming the value of δ_{\max} occurs at the center of the wall or $L/2$, Eq. (3) is reduced to:

$$[4] \quad 0 = \operatorname{erfc} \left[\frac{\left(\frac{L}{2} - A \right)}{B} \right]$$

The complementary error function approaches zero as x increases to infinity. A reasonable truncation must be made to evaluate the function. Based on least-squares fitting of the movement data by the Levenberg-Marqson procedure (Gill et al. 1981), it is assumed that the complementary error function is essentially zero when

$$[5] \quad \left[\frac{\left(\frac{L}{2} - A \right)}{B} \right] \approx 2.8$$

Therefore, the value of B can be calculated for any given A as:

$$[6] \quad B \approx \left[\frac{\left(\frac{L}{2} - A \right)}{2.8} \right]$$

In summary, the fitting parameters for the erfc illustrated in Figures 1 and 2 are:

A = the inflection point of the erfc; occurs at the location where the movement is $\delta_{\max}/2$

$$B \approx \left[\frac{\left(\frac{L}{2} - A \right)}{2.8} \right]$$

$C = \delta_{\max}/2$, opposite in sign to D

$D = \delta_{\max}$, same sign as the fitted data

x = distance measured from corner of excavation

The fitting function is then expressed as:

$$[7] \quad \delta(x) = \delta_{\max} \left(1 - \frac{1}{2} * \operatorname{erfc} \left(\frac{x - A}{B} \right) \right) = \delta_{\max} \left(1 - \frac{1}{2} * \operatorname{erfc} \left(\frac{2.8(x - A)}{0.5L - A} \right) \right)$$

Boscardin and Cording (1989) and Burland and Wroth (1974) have related angular distortion to the magnitude of damage in buildings affected by ground movements. Angular distortion is the distortion (defined as the slope of the ground surface settlement profile) minus the rigid body rotation of an affected structure. One can calculate the distortions parallel to the wall directly from the erfc equation. For example, the slope of the line tangent to the inflection point, A (see Figure 2), represents the maximum slope of the settlement profile (or distortion) and is written as:

$$[8] \quad \text{Maximum slope} = \frac{-\delta_{\max}}{B\sqrt{\pi}} = \frac{2.8\delta_{\max}}{(0.5L - A)\sqrt{\pi}}$$

The calculation of the maximum slope based on Eq. 8 is affected by the selection of the factor of 2.8 as a truncation of the erfc function. The adequacy of this selection will be

discussed later when comparing the distortions calculated based on the erfc fitting distribution to those calculated from the ground movements measured in the field.

DEVELOPMENT OF ERFC BASED ON OPTICAL SURVEY DATA FROM THE LURIE CENTER EXCAVATION

The ability of the erfc to adequately represent ground movements is demonstrated with data collected at the 12.8 m deep excavation for the Robert H. Lurie Medical Research Building (Finno and Roboski 2005). As shown in Figure 4, the excavation was made through 4 m of loose to medium dense granular fill, 5 m of medium dense to dense beach sand, and 3.8 m of soft to medium clay. A stiff clay layer was encountered 4 m below the bottom of the cut. A PZ-27 sheet pile wall supported the excavation. Three levels of tiebacks provided lateral support, except along the east wall where two levels of tiebacks were installed due to the presence of the basement for the adjacent Prentice Pavilion. Anchors were preloaded after proof and performance testing by locking off loads at 100% of their design values of 623, 534 and 632 kN for levels 1 through 3, respectively. Gas, water, and sewer utilities were located in the three streets bounding the site.

Figure 5 shows the locations of the 200 monitoring points that were established along the north, west and south walls to evaluate whether the ground movements around the excavation were within tolerable limits established primarily to protect the utilities. The monitoring points included embedded anchors set about 1 m into the soil, P.K. nails set into the adjacent asphalt roadbeds, and utility monitoring points consisting of rods

isolated from the surrounding soil by casing set atop adjacent gas, water and sewer mains. The monitoring points are divided into rows of equal distance from the excavation to evaluate the fit of the erfc. As seen in Figure 5, both the north and west walls have two distinct rows of optical survey points that are aligned parallel to the wall, while three rows parallel the south wall. A letter—F, G, or H, indicates each row. The optical survey points and eight inclinometers were read on a weekly basis during the excavation. Finno and Roboski (2005) provide a detailed description of the performance of the excavation support system during excavation.

Seven major construction stages are defined in Table 1, including wall construction, excavation and tie-back installation, and backfilling between the temporary support wall and the permanent basement wall. Significant ground movements occurred for stages 4, 5, and 6 as the excavation was cut through the clays. The erfc parameters and corresponding fitting functions were generated for ground movements associated with each of these stages. Values of A and δ_{\max} are required to compute Eq. (7) for each stage; the A parameter depends on the length, L , and the height of the excavation, H_e , at each particular stage, and the maximum movement, δ_{\max} is a measured value.

Ground movements (both settlement and lateral) for the heights of excavation defined by stages 4, 5, and 6, for the optical points along the north, west, and south walls were used to compute values of the inflection point, A , via the procedure described in Figure 3. Figure 6 shows these values of A plotted versus distance behind the wall for each stage. As suggested by these data, A is approximately constant with distance behind the wall, but decreases with increasing depth of excavation. Hence, the same A value is used for all distances behind the wall for each stage, only the δ_{\max} values change as a

function of distance behind the wall. Furthermore, the value of A decreases with excavation depth for a given length of wall likely due to the end restraints which become more pronounced as the excavation deepens.

Figure 7 plots the resulting relationship of A normalized by one-half the wall length versus the height of excavation normalized by the length of wall for each stage. By trial and error, a natural log decay curve was fit to the dimensionless relationship which illustrates that A decreases with greater normalized height of excavation:

$$[9] \quad \frac{2A}{L} = -0.069 * \ln\left(\frac{H_e}{L}\right) - 0.03$$

This expression was derived for normalized height values between 0.085 and 0.22. As will be subsequently discussed, examination of data from other excavations extends this relationship to normalized height values as large as 0.93. Substituting Eq. (9) into Eqs. (7) and (8) yields expressions for the fitting function and maximum distortion, respectively, in terms of excavation geometry and δ_{\max} only.

$$[10] \quad \delta(x) = \delta_{\max} \left(1 - \frac{1}{2} * \operatorname{erf} \left(\frac{2.8(x + L [0.015 + 0.035 \ln \frac{H_e}{L}])}{0.5L - L [0.015 + 0.035 \ln \frac{H_e}{L}]} \right) \right)$$

$$[11] \quad \text{Maximum Distortion} = \frac{2.8\delta_{\max}}{0.5L \left(0.97 + 0.069 \ln \frac{H_e}{L} \right) \sqrt{\pi}}$$

EVALUATION OF ERFC FIT AT LURIE PROJECT

Movements behind the south wall

The erfc fits for rows of optical survey points along the south wall of the Lurie Center at stages 5 and 6 were computed from Eq. (10) based on appropriate values of H_e , L and the observed maximum ground movement. Figure 8 shows the settlements and Figure 9 shows the lateral ground movements and the calculated erfc fit for each stage and each row of data.

A measure of the “goodness of fit” is given by the standard deviation of the residual error. Phoon and Kulhawy (1999) suggested that the fluctuations of detrended data about a central value can be evaluated by the standard deviation of the sum of residuals squared. Assuming the erfc distribution is the mean of the optical data points, this definition can be applied to the erfc fits herein. The standard deviation of the erfc is written as:

$$[12] \quad SD_{ERFC} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n [\delta_{ERFC_i} - \delta_{OBS_i}]^2}$$

where the residual $[\delta_{ERFC} - \delta_{OBS}]$ is the difference in the erfc-calculated and measured ground movement values, and n is the number of data points. The standard deviation can be normalized by the maximum calculated value, resulting in a percentage of error proportional to the maximum measured movement such that:

$$[13] \quad \% \text{ Error} = \frac{SD_{ERFC}}{\delta_{\max}} * 100\%$$

The smaller the percentage of error is, the better the fit. The error value does not reflect the confidence intervals of the parameters used to calculate the erfc fit, rather it is simply

a measure of how closely a trend fits actual data points, and will be used herein to indicate the quality of the erfc fit.

Table 2 lists the error values for stages 5 and 6, when the excavation was advanced through the clays at the site. The function was not fit to data collected at earlier stages because they were related to site preparation, caisson installation for the foundations for the building, and excavation through the upper granular soils. It can be seen that the values of error for fits developed at stage 6 are reasonably low, indicating a good fit. Ground movement measurements at stage 5 exhibited more scatter than those at stage 6, as indicated by higher error values. For the most part, the fit is slightly better for the settlement data. Note that these errors also reflect uncertainties in the optical survey data wherein accuracies can be expected to be no more than ± 4 mm and in the difference in the types of monitoring points employed at the site. No significant differences between the settlements obtained with the P.K. nails, embedded settlement points and utility monitoring points were observed for stage 5 and 6 data (Finno et al. 2003).

Movements behind the north and west walls

Occasionally, the presence of adjacent utilities and structures may cause the location of the maximum ground movement to be shifted from the middle point of the wall. An example of an off-centered distribution is the ground movement distribution along the north wall of the Lurie Center. The settlements behind the north wall after stage 5 of construction are shown in Figure 10 plotted versus distance from the west corner of the excavation. The excavation limits are indicated in addition to the limits of a

timber pile supported pedestrian tunnel located outside the excavation walls (see Figure 5). The timber pile foundation adjacent to the excavation walls limited the ground movements in the area behind the tunnel. In this case the maximum movements occurred about 60 m from the excavation corner. To fit the erfc to the movements along this wall, the movements which occurred within the limits of the timber pile supported tunnel were ignored, and a new “corner” was set at the edge of the tunnel, thus reducing the length of the wall from 88 meters to 64 meters. This new “wall length” is then used to define the fitting function Eq. (10). Figure 10 shows the calculated erfc fits at stages 5 and 6 for the two parallel rows of settlement points along the north wall. Table 3 summarizes the error values for the calculated fits for all rows behind the North and West walls. As with the South wall, the fits tend to be better for stage 6. In general, the error values for the lateral ground movements are comparable to those for the settlements.

Computed distortions

Equation 11 can be used to calculate the maximum slope of the settlement profile (distortion) parallel to the excavation walls given the length of the excavation wall or the length along which the distribution is being calculated, the depth of the excavation and the maximum movement. Figure 11 summarizes the settlement as a result of excavation to stages 5 and 6 along the south wall. In addition, the maximum distortions parallel to the wall were calculated according to Eq. 11 and plotted at the point of inflection to compare with the field observations. The calculated distortions reasonably represent the *in situ* distortions upon visual inspection, particularly at the full excavated depth of stage

6. In addition, the erfc calculated distortion values are comparable to those found from contoured settlement data reported by Finno and Roboski (2005), where the distortions parallel to the excavation wall were found to be as significant as those perpendicular to the site.

APPLICATION TO OTHER SITES

The ability of the erfc to fit ground movements parallel to an excavation under more general conditions is illustrated by comparing the erfc fitting function based on Eq. (10) to settlement data from three projects reported in literature. As summarized in Table 4, these excavations occurred in varying soil conditions and with different support systems. The Taipei National Enterprise Center excavation was supported by a diaphragm wall and was excavated through alternating layers of silty clay and sand. The Tokyo excavation reported by Takagi et al. (1984) was lowered through layers of peat, silt, and fine sand. The Chicago and State subway expansion excavation was supported by a secant pile wall and was made through a surficial urban fill and clays of increasing shear strength.

Taipei National Enterprise Center

Ou et al. (2000a, 2000b) described a 19.7 m deep excavation in Taiwan. The approximately trapezoidal-shaped excavation was supported by a 90 cm thick diaphragm wall constructed to a depth below ground surface of 35 m. The excavation was completed using the top-down construction method with concrete floor slabs serving as lateral support for the diaphragm wall. The site stratigraphy consisted of 46 m of

alternating silty clay and silty sand layers overlying a thick gravel formation. The groundwater table was located at 2 m below the ground surface prior to excavation. The excavation behavior was most significantly affected by a 25 m thick layer of lightly overconsolidated silty clay with a ratio of undrained shear strength to effective overburden pressure of 0.32 to 0.34, based on field vane shear tests results.

Buildings surrounded most of the site. A dense array of settlement monitoring points was installed in the section with no adjacent structures along an approximate 36-meter length of the 107.2 m south wall of the excavation. Measurements reported exclude movements due to wall construction.

Figure 12 compares the settlement data and the erfc fit calculated with Eq. (10) using the geometry of the excavation and δ_{\max} for three depths of excavation, 15.2 m, 17.3 m, and 19.7 m; and for three distances from the wall, 5 m, 11 m, and 19 m. The best fit was obtained with settlement measurements from a distance of 11 m away from the excavation, the distance from the wall at which the maximum settlement was observed. The data clearly show the three-dimensional effects due to the excavation corners. The error values for each of the calculated fits are also shown in Figure 12. It can be seen that the highest value of error is 15%, indicating that a reasonable fit was found by using the above procedure. Higher error values farther from the excavation wall may be due to the presence of surrounding buildings. The A -values calculated given Eq. (9) are also shown.

Tokyo Excavation

Takagi et al. (1984) presented settlements measured on two survey lines located 1.5 m and 3 m away from an excavation in Tokyo. The stratigraphy at the site consisted of two main layers, 10 m of silt overlying fine sand, separated by a thin layer of peat.

Blow counts increased from zero in the peat layer at 10 m below the ground surface, to a maximum of 50 in the sand layer located 14 m below the ground surface. The settlement data were taken adjacent to the 30 m long south wall. Data were reported corresponding to excavation depths of 20 and 28 m. The support system for the excavation was not described in the article.

Figure 13 presents the settlement data and corresponding calculated fit for two excavation depths. Settlement data are plotted versus distance to the closest corner. Given the normalized heights of excavation, 0.67 and 0.93, Eq. (9) was used to calculate the values of A shown in Figure 13. The maximum settlement value was selected from the data. The error values for each fit are also shown in Figure 13. The highest value of error calculated for these four fits is 11%, indicating that the proposed procedure reasonably calculates the observed settlement distribution parallel to the excavation.

Chicago-State Subway Excavation

Finno and Bryson (2002) described a 12.2 m deep excavation adjacent to a three-story school supported on shallow foundations. The stratigraphy consisted of 4.3 m of sand and fill overlying a thin stiff crust layer. Below the stiff clay crust was a sequence of layers of increasingly stiff clays. The excavation bottomed out in a medium stiff clay layer. The excavation support system consisted of a 0.9 m wide secant pile wall supported laterally by both cross-lot bracing and tiebacks. Steel pipe struts placed at a horizontal spacing of 6.1 m formed the first level of support. Re-groutable tieback anchors were used for the second and third levels of support, placed at a 1.5 m center-to-center spacing. The bonded zone of the anchors was located in a stiff clay layer. Settlements were monitored using optical survey points on interior columns and exterior

walls of the adjacent school. The interior column survey points were placed in the basement. The exterior survey points were established approximately 1 m above grade. Figure 14 plots the observed settlements and calculated erfc fit along the east wall of the excavation for two rows of optical survey points. The optical survey points are 3.9 m and 15 m behind the wall of the excavation. The length of excavation at the time this data was collected was 47.3 m. Data for two heights of excavation are reported, 5.5 m and 12.2 m. From Eq. (9), A is calculated for each height of excavation. The calculated fits are shown in Figure 14. The “goodness of fit” measure and values of A for each data set are also shown. The low error values indicate a reasonable fit, especially at final excavation depth.

Summary

As shown by these three data sets, the success of the erfc fitting of ground movement data does not depend on stratigraphy or the excavation support system, at least for the cases considered herein. It has been shown to reasonably predict distributions of ground movements arising from excavations made through a variety of surface conditions and support systems. Thus, the erfc fitting procedure is applicable to conditions other than those found at the Lurie excavation site, from which the procedure was derived. The influence of differing soil conditions is likely accounted for in the magnitude of the ground movements. The erfc fitting procedure only provides a distribution of ground movements parallel to the excavation; the magnitude of the expected maximum ground movement must be estimated separately.

The error values calculated for the erfc fits for the three case studies in this section generally are lower than those calculated for the fitting of the Lurie ground

movement data. This is likely due to the fact that very shallow excavations were made to replace utilities at the Lurie site; these activities impacted the adjacent settlement points in a random pattern.

Further verification of the relationship described by Eq. (9) is given by calculating the locations of the inflection points, A , for each of the three excavations described in this section according to the procedure summarized in Figure 3. The results are shown in Figure 15. This figure shows the same equation as Figure 7, but includes the three cases in addition to the Lurie data such that a wider range of H_e/L is investigated. As can be seen, the calculated locations of inflection points for the three additional excavations support the validity of the natural log decay fit, Eq. (9), found with the Lurie data. The data from the excavation described by Takagi et al. (1984) supports the assumption that as H_e/L nears 1, the location of the inflection point will occur outside the limits of the excavation wall. This method is supported by data collected at excavation stages when $0.1 \leq H_e/L \leq 0.9$. However, more case studies are needed to verify these trends, especially at H_e/L ratios greater than 0.3.

RECOMMENDED PROCEDURE

The following procedure is recommended to predict the distribution of ground movements parallel to the excavation wall with the complementary error function.

- 1) Employ a semi-empirical method to calculate either the maximum lateral movement or settlement expected for the given support system, geometry of excavation and soil conditions (e.g., Peck 1969; Clough et al. 1989; Clough and

- O'Rourke 1990). Alternately the maximum lateral movements at the wall can be estimated from the results of a finite element analysis. This value of maximum movement should correspond to the largest value of movement at the center of the excavation.
- 2) Find the maximum movement for the distance from the wall that is of interest, using methods described by Hsieh and Ou (1998). This value corresponds to δ_{\max} of Eq. (10).
 - 3) Given the maximum ground movement and the length and depth of excavation, compute the distribution of movements with Eq. (10). This fit is mirrored about the centerline of the excavation to give a distribution along the full wall.
 - 4) Calculate distortions expected parallel to the excavation in cases where critical utilities and other structures may exist. Maximum distortions will occur at the inflection point and can be determined from Eq. (11).

COMMENTS

A limitation of the erfc fitting method as developed herein is that it only accurately represents field conditions where excavation-induced ground movements can develop with little restraint. For example, it can be used to estimate movements of utilities when the utilities move with the ground, as usually occurs when the diameter of the utility is relatively small (Finno et al. 2003). When the affected utility is a larger diameter pipe such as a water main, the pipe will provide restraint for the movements, and in this case, the erfc fitting procedure will provide a conservative estimate of the movements.

The procedure can be used to compute the settlements of buildings with reasonable accuracy when the building adjacent to the excavation is supported on shallow foundations, as is common with many older, smaller structures in urban areas. However, the erfc fitting procedure should not be used to estimate lateral movements in the case of buildings with stiff floor systems that provide lateral restraint that has been shown to restrict the lateral movement of the building (e.g., Finno and Bryson 2002).

If adjacent buildings are supported on deep foundations or if there are deep foundations between the building and the excavation, then the restraint provided by the deep foundations will affect the distribution of movements. The erfc fitting function as developed herein would not be applicable. The fitting procedure was unsuccessfully applied by the authors to a deep excavation in Boston (Bono et al. 1992) for which the ground movement data were measured at the column locations of an adjacent building. The affected building was supported on deep caissons which did not extend below the bottom of the excavation. The restraint afforded by the caissons affected the distributions of the ground movements. Another example of restraint provided by piles between the excavation and the affected building is the situation described in this paper along the north wall of the Lurie project due to the presence of the timber pile-supported pedestrian tunnel. Similar situations could also arise in urban areas from abandoned deep foundations. Note that the difference in the erfc fit calculated assuming free-field conditions and the actual settlement profile may be used as a measure of the soil-structure interaction.

When using the erfc to calculate an expected distribution of settlement before or during an excavation, the distribution will only be as good as the semi-empirical

approximation made of the maximum ground movement. This maximum ground movement is assumed to occur at the center of the wall, except when ground movements are restrained by existing foundations and utilities. The calculated distribution is symmetric about the chosen center point.

CONCLUSIONS

Based on the results of analyses of performance data and the complementary error function presented herein, the following conclusions can be drawn:

- 1) The distribution of ground movements parallel to an excavation can be represented adequately by a fitting function based on the complementary error function when the excavation induced ground movements can develop with little restraint.
- 2) Computed ground movement profiles based on the complementary error function depend on the length and height of excavation and the maximum ground movement. Distributions of both settlements and lateral ground movements can be estimated with this function. The proposed relationship has been shown to be applicable to four excavation cases where sufficient ground surface movement data were reported to adequately describe the distribution.
- 3) Distortions which develop parallel to an excavation can be calculated given the erfc distribution for an excavation.

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List of Tables

Table 1. Construction stages

Table 2. Error values: south wall for stages 5 and 6

Table 3. Error values: north and west walls for stages 5 and 6

Table 4. Summary of projects where the erfc was evaluated

Table 1. Construction stages

Construction stage	Summary of major activity
1 through 4	Install PZ-27 sheeting after potholing; excavate and install first level tiebacks; construct caissons; excavate and install second level tiebacks
5	Excavate to – 5.8 m CCD; install third level tiebacks at -5.2 m CCD
6	Excavate to final grade at – 8.5 m CCD; construct grade beams
7	Construct basement walls and slab; backfill space between sheeting and basement wall

Note: CCD refers to Chicago City Datum.

Table 2. Error values: south wall for stages 5 and 6.

Stage	Distance behind wall (m)	A	Settlement % error	Lateral movement % error
Stage 5 $H_e = 10.1$ m	6.7	4.4	29	34
	8.8	4.4	34	33
	12.8	4.4	38	38
Stage 6 $H_e = 12.8$ m	6.7	3.8	16	16
	8.8	3.8	22	16
	12.8	3.8	13	17

Table 3. Error values: north and west walls for stages 5 and 6.

Stage	North wall			West wall		
	Distance behind wall (m)	Settlement % error	Lateral movement % error	Distance behind wall (m)	Settlement % error	Lateral movement % error
Stage 5	5.2	20	15	4	13	14
$H_e = 10.1$	8.2	27	21	7.6	28	12
Stage 6	5.2	17	15	4	8	14
$H_e = 12.8\text{m}$	8.2	17	13	7.6	24	16

Table 4. Summary of projects to which erfc applied.

Case	Support System	Geometry		Soil Stratigraphy	Strength Measure		Support Stiffness	3D Effect Ratio	Comments
		He (m)	Plan View		N^1	FS_{BH}^2	$EI/h^4\gamma_w$	$\frac{\delta_{corner}}{\delta_{center}}$ Settlement	
Taipei National Enterprise Center (TNEC) Taipei, Taiwan Ou et al. (2000)	Diaphragm Wall (900 mm, 35 m depth); Top Down construction, 5 level of floor slabs	19.7	107 m x 45 m	Five layers alternating silty clay and silty sand with increasing shear strength overlying gravel formation 46 m below ground surface	N/A	1.6	1343	0.38	Retaining wall supported by the concrete floor slabs.
Takagi et al. (1984) Tokyo, Japan	Support system not indicated	28	30 m x ??	1.5 m topsoil; 8.5 m silt; 1.5 m peat; 8 m sand; 3 m silty sand; fine sand	18-39 (11.5-12.5) 39-49 (12.5-14) 49 (14-30)	N/A	N/A	0.6	Excavation reported as part of study on excavation effects on adjacent utilities.
Chicago and State Subway Expansion Chicago, IL Finno and Bryson (2002)	Secant pile wall (900 mm, 18.3 m depth); One level cross-lot, two level tieback	12.2	24 m x 47 m	Fill deposit overlying clays of increasing shear strength: 4.2 m sand and fill; 1.75 m clay crust; 5 m soft clay; 6 m medium clay; 4 m stiff clay; hard clay	N/A	1.4	2125	0.45	Two story building with one basement level on shallow foundations adjacent to excavation.

Note: 1. SPT value indicated for 30 cm penetration for depth (meter) below ground surface indicated in parentheses.

2. FS_{BH} - factor of safety versus basal heave.

List of Figures

Figure 1. Complementary error function

Figure 2. Derived fitting parameters for the complementary error function

Figure 3. Example of finding erfc fitting parameters with settlement data

Figure 4. Subsurface conditions and support system at the Lurie Center excavation

Figure 5. Plan view of the Lurie Center excavation site

Figure 6. A versus distance behind wall for Lurie Center excavation

Figure 7. $2A/L$ vs. He/L for Lurie Center data

Figure 8. South wall—observed and calculated settlements for Lurie Center data

Figure 9. South wall—observed and calculated lateral movements for Lurie Center data

Figure 10. Effects of pile-supported tunnel on erfc fit at north wall

Figure 11. Comparison of distortions calculated from erfc fit and the settlement data for the south wall.

Figure 12. Observed and calculated settlements behind an excavation wall in Taipei (Ou et al. 2000)

Figure 13. Observed and calculated settlements 1.5 and 3 m behind an excavation wall in Tokyo (Takagi et al. 1984)

Figure 14. Observed and calculated settlements 3.9 and 15 m behind an excavation wall in Chicago (Finno and Bryson 2002)

Figure 15. $2A/L$ vs. He/L for Lurie Center and three additional sites