Sensor Coverage and Location for Real-Time Traffic Prediction in Large-Scale Networks

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The ability to observe flow patterns and performance characteristics of dynamic transportation systems remains an important challenge for transportation agencies, notwithstanding continuing advances in surveillance and communication technologies. As these technologies continue to become more reliable and cost-effective, demand for travel information is also growing, as are the potential and the ability to use sensor and probe information in sophisticated decision support systems for traffic systems management. This paper focuses on improving the efficiency of data collection in transportation networks by studying how sensor placement affects network observability. The objective of this study is to identify a set of sensor locations that optimize the coverage of origin–destination demand flows of the road network and maximize the information gains through observation data over the network, while minimizing the uncertainties of the estimated origin–destination demand matrix. The proposed sensor models consider problems where the numbers of sensors are limited and unlimited. The paper also provides several examples to illustrate the relative effectiveness of the proposed methodologies.

The ability to observe flow patterns and performance characteristics of dynamic transportation systems remains an important challenge for transportation agencies, notwithstanding continuing advances in surveillance and communication technologies. As these technologies continue to become more reliable and cost-effective, demand for travel information is also growing, as are the potential and ability to use sensor and probe information in sophisticated decision support systems for traffic systems management. Whereas probe data based on cellular-assisted Global Positioning System and other cellular phone technologies hold the promise of near-ubiquitous information coverage in a network, measurements on system state at given locations using fixed sensors remain the backbone of most traffic management centers for traffic management and control purposes. Given the deployment and maintenance costs of such installations, most agencies are called on to determine the number and locations of such sensors across a given network.

To improve the efficiency of data collection in transportation networks, it is critical to understand how sensor placement affects network observability. Furthermore, a new generation of real-time network traffic estimation and prediction systems is designed to interact with real-time sensor data to support system management decisions and through estimation, prediction, and control generation cycles (*1*). For example, real-time dynamic traffic assignment (DTA) systems such as Dynasmart-X and DynaMIT use sensor measurements on a subset of the network links as a basis for estimating and predicting traffic conditions on a quasi-continuous basis. In particular, the sensor measurements are combined with current values and historical information to estimate prevailing origin–destination (O-D) patterns and predict their near-term evolution, in addition to predicting the network traffic patterns associated with these O-D demands. The objective of this study is to identify a set of sensor locations that optimize the coverage of O-D demand flows of the road network and maximize the information gains through observation data over the network, while minimizing the uncertainty in the estimated O-D demand matrix.

This paper is composed of six sections. The second section provides a review of early research on the sensor location problem. The third section presents a framework for approaching the sensor location problem and discusses models that can be used for the cases of unlimited and limited numbers of sensors. The fourth section includes an analysis that illustrates the information gains and trade-offs associated with various sensor location schemes. The fifth section examines the results produced using the proposed models. The final section concludes the paper and delineates some areas of future work.

BACKGROUND

The growing need of agencies to obtain real-time information on the traffic state of key facilities in the systems they manage is driving interest in cost-effective deployment of sensor technologies across the networks they manage. This has led to greater interest in the sensor location problem. Understanding the trade-offs between sensor investments and information gain is critical to the agencies' decision making in this regard. A number of researchers have addressed limited versions of the sensor location problem, primarily in the context of O-D matrix estimation using link counts from road sensor stations.

The past two decades have seen development and application of several approaches for the O-D matrix estimation problem. In general, these approaches fall into two categories: traffic-assignment-based approaches and statistical inference approaches.

The first category includes "information minimization" (entropy maximization) models. Van Zuylen and Willumsen (*2*) developed two models based on information minimization and entropy maximization principles to estimate an O-D matrix from traffic counts that seeks to reproduce the observed link flows. Fisk (*3*) combined Van Zuylen and Willumsen's (*2*) maximum entropy model with a user-equilibrium

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model into a single mathematical problem in a bilevel programming formulation. Recognizing that the number of O-D pairs (unknown variables) in the O-D estimation problem is normally greater than the link traffic stations, it has become common practice to integrate the a priori O-D matrix with the link counts to identify a unique estimated O-D matrix. The second category includes maximum likelihood (ML) approaches, generalized least-squares (GLS) approaches, and a Bayesian inference approach. Spiess (*4*) assumed the O-D demand to be a realization from independent Poisson distributions with unknown means. A ML model was formulated to estimate these means to reproduce the estimated link flows consistently with the observed flows. Cascetta (*5*) proposed a GLS estimator combining with traffic counts via an assignment model. Bell (*6*) presented an algorithm for the constrained GLS problem and established its convergence. Maher (*7*) assumed that the prior O-D matrix and the observed link counts follow multivariate normal distributions and proposed a Bayesian statistical approach to update the prior O-D matrix. More recently, new sources of information produced by emerging technologies, such as automatic vehicle identification, have been used to estimate the O-D matrix with point sensor data (*8–10*).

Aware of the inherent connection between the O-D estimation problem and link count observations, several researchers have approached the sensor location problem as an O-D covering problem. Lam and Lo (*11*) proposed traffic flow volume and O-D coverage criteria to determine priorities for locating sensors. By using a concept of maximum possible relative error (MPRE) to bound the actual relative error, Yang et al. (*12*) formulated a simple quadratic programming problem and showed that, if an O-D pair is not covered by a sensor, the MPRE is infinite. On the basis of the MPRE, Yang and Zhou (*13*) proposed four basic rules for the sensor location problem:

Rule 1. O-D covering rule: A certain portion of trips between any O-D pair should be observed.

Rule 2. Maximal flow fraction rule: For a particular O-D pair, link with the maximal fraction of that O-D flow should be selected.

Rule 3. Maximal flow-intercepting rule: Under a certain number of sensors constraint, the maximal number of O-D pairs should be observed.

Rule 4. Link-independence rule: The resultant traffic counts on the selected links should not be linearly dependent.

Yim and Lam (*14*) evaluated the maximal net O-D capture rule and the maximal total O-D capture rule on a large-scale network. Bianco et al. (*15*) proposed an iterative two-stage procedure that focuses on maximizing "coverage" in terms of geographic connectivity and size of the O-D demand population. Chootinan et al. (*16*) formulated a bi-objective model for locating traffic counting stations for the purpose of O-D matrix estimation. They considered Yang and Zhou's (*13*) maximal covering rule while minimizing the number of sensors as two conflicting criteria and proposed a multiobjective method to obtain a good compromise solution. Ehlert et al. (*17*) extended Yang and Zhou's (*13*) work, taking the existing sensors into account and thereby seeking a second-best solution. Yang et al. (*18*) extended their work to the screenline-based sensor location problem and formulated an integer linear programming model. Pravinvongvuth et al. (*19*) proposed a methodology for selecting the most preferred plan from the set of Pareto optimal solutions obtained from solving a multiobjective automatic vehicle identification reader location problem constrained by the resource limitation as well as the O-D flow coverage. The previously mentioned approaches were all proposed or implemented under the assumption of error-free measurements, where the objective is to maximize O-D coverage. None of these studies considered reducing the uncertainty in the O-D matrix estimates.

Eisenman et al. (*20*) proposed a conceptual Kalman-filtering-based framework to maximize the information gain and minimize the error of the estimated O-D demand matrix to find sensor locations. They used a simulation-based approach to evaluate the value of various sensor location schemes for real-time network traffic estimation and prediction applications in a large-scale network. Zhou and List (*21*) focused on locating a limited number of traffic-counting stations and automatic vehicle identification readers in a network to maximize expected information gain for the subsequent O-D demand estimation problem solution.

The approach presented here seeks to identify a set of sensor locations that optimize the coverage of O-D demand flows of the road network and maximize the information gains through observation data over the network, while minimizing the uncertainty in the estimated O-D demand matrix. It stands apart from most approaches in the literature in that it explicitly considers time-varying O-D demand.

FRAMEWORK

This section presents methodologic approaches to two variants of a so-called sensor location problem. The first methodology is focused on solving the sensor location problem with an unlimited number of sensors (unconstrained). The second methodology is focused on solving the sensor location problem with a limited number of sensors (constrained).

Unlimited Network Sensors

Yang and Zhou (*13*) formulated a binary integer program to determine the minimum number of sensor locations required to satisfy an O-D covering rule for a road network with a given prior O-D matrix and path selection.

$$\text{minimize} \sum_{a \in A} z_a$$

subject to

$$\sum_{a \in A} \delta_{aw} z_a \geq 1 \qquad w \in W$$

$$z_a = 0,\ 1 \qquad a \in A$$

where $z_a = 1$ if a sensor is located on link a and is 0 otherwise, and $\delta_{aw} = 1$ if some trips between O-D pair w pass over link $a \in A$ and 0 otherwise. It can be shown that the resultant sensor location solution satisfies the O-D covering rule and that selected links will be independent. A large network containing many O-D zones and a significant number of links may be difficult to solve with this formulation. A heuristic used to solve the proposed formulation might find only a set of feasible or suboptimal solutions instead of the optimal set. This is due to the trade-off between computation time and solution quality. In addition, Yang and Zhou's model is based on static traffic assignment and considers an O-D pair covered once a sensor is located on a single link of the paths between that particular O-D pair. In reality, the path set between O-D pairs evolves with time of day. Thus, Yang and Zhou's O-D covering model does not guarantee a result in which all O-D pairs are covered at all times throughout the day.

O-D Covering Problem with Time-Varying Network Flows

To account for sensor location problems on large-scale networks with time-varying flows (e.g., determined with DTA methodology), a method is proposed that considers a time-varying path determinant. This model will result in a set of sensor locations on the links along the paths covering a subset of O-D pairs that experience O-D demand flows in excess of a minimum number of trips, ζ^τ, where ζ^τ is a threshold termed as the " degree" that defines the relevant O-D pairs in any time interval. The following binary integer program formulation of the sensor location problem (SLP) is presented to provide coverage of the O-D pairs with a flow beyond a preset "relevant degree" ζ^τ.

SLP-1 minimize $\sum_{a\in A} z_a^\tau$

subject to

$$\sum_{a\in A} \delta_{aw}^\tau z_a^\tau \geq 1 \qquad w \in W, \text{ with } d_w^\tau \geq \zeta^\tau, \tau \in T$$

$$z_a^\tau = 0, 1, a \in A$$

$$\delta_{aw}^\tau = \text{assignment } \lfloor d_w \rfloor \text{ from DTA}, a \in A, w \in W, \tau \in T$$

where $z_a^\tau = 1$ if a sensor is located on link a during departure time τ and 0 otherwise, and $\delta_{aw}^\tau = 1$ if some trips with departure time τ between O-D pair w traverse link $a \in A$ and 0 otherwise. T is the planning horizon for sensor data collection.

Algorithm

Step 0. Run DTA simulation software [Dynasmart-X (*22*) in this study] given a prior O-D demand matrix to get δ_{aw}^τ, $a \in A$, $w \in W$, $\tau \in T$, $\tau = \tau_0$.

$$\zeta^\tau = \zeta_0^{\tau_0}$$

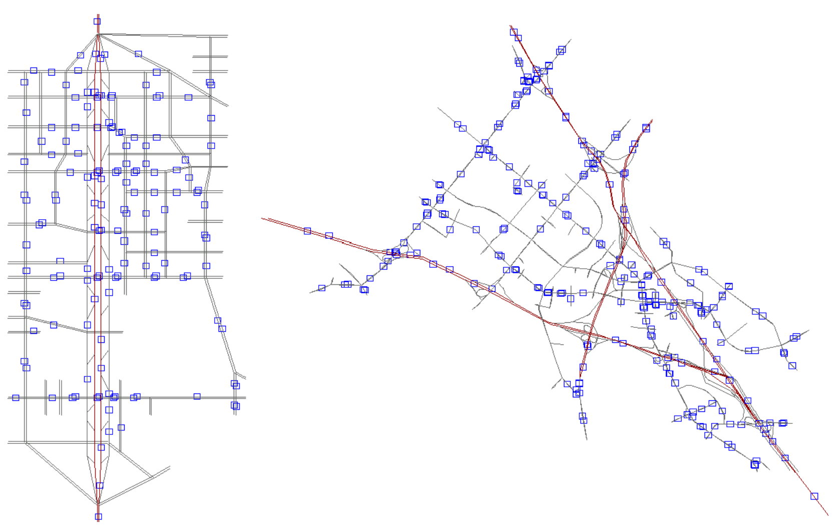
Step 1. If $\tau < T$, filter out the O-D pairs with flow less than ζ^τ. Run branch-and-bound (BnB) method to solve the binary integer model to obtain the solution path set z^τ of SLP-1. Otherwise, if $\tau \geq T$, $Z = \bigcup_{\tau\in T} \{z_a^\tau\}$, stop.

Step 2. Set $\tau = \tau + 1$, ζ^τ to satisfy the O-D coverage percentage in time interval τ; go to Step 1.

To illustrate the proposed model, Figure 1 shows the sensor locations for two networks: Fort Worth, Texas, with 147 sensors that cover 156 O-D pairs [13 traffic analysis zones (TAZs)], including 180 nodes and 445 links, and Irvine, California, with 238 sensors that cover 3,660 O-D pairs (61 TAZs) , including 326 nodes and 626 links. The a priori relevant degree $\zeta^\tau = 0$ under the DTA. The time period of interest is the morning peak from 6:30 to 8:30 a.m. Figure 2 presents the solution results for the static model proposed by Yang et al. (*18*). The same networks using static information result in having 12 sensors and 44 sensors, respectively.

The results of the dynamic model show that, to cover each O-D pair in the network across time, more sensors are needed than those obtained by solving the sensor location problem based on static traffic assignment. Figure 3 shows the minimum number of required sensor locations for each departure time interval τ over the analysis horizon.

Figure 1 shows the sensor locations on two real traffic networks covering network O-D pairs in different departure time intervals from 6:30 to 8:30 a.m. Although the sensor locations found by the algorithm for SLP-1 can cover all the O-D pairs across time, there might exist more than one sensor covering the same O-D pair because the proposed problem has two dimensions: temporal and spatial. Instead of considering the sensor locations for each time period separately, the SLP-1′ model integrates the constraint conditions during different departure intervals into one constraint set and solves the binary integer model using the BnB algorithm once the simulation assignment is completed and the resulting routing policies become available in all the departure time intervals. Because it is assumed in this section that the number of sensors is unlimited, only solutions to model SLP-1 are analyzed.



(a) (b)

FIGURE 1 Sensor locations by DTA in (*a*) Fort Worth network and (*b*) Irvine network.

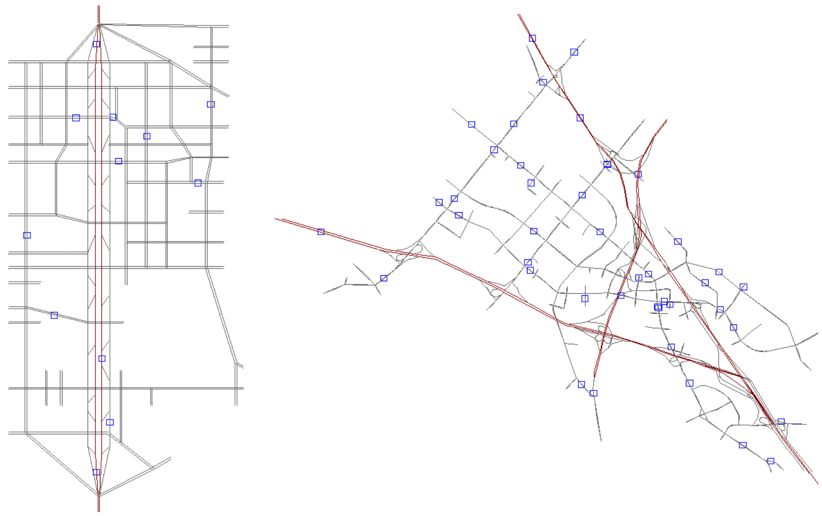


FIGURE 2 Sensor locations by static model in (*a*) Fort Worth network and (*b*) Irvine network.

Model SLP-1′ is formulated as follows.

$$\text{SLP-1}' \quad \text{minimize} \sum_{a \in A} z_a$$

subject to

$$\sum_{a \in A} \delta_{aw}^{\tau} z_a \geq 1 \qquad w \in W, \text{ with } d_w^{\tau} \geq \zeta^{\tau}, \tau \in T$$

$$z_a = 0, 1, a \in A$$

$$\delta_{aw}^{\tau} = \text{assignment } \lfloor d_w \rfloor \text{ from DTA}, a \in A, w \in W, \tau \in T$$

where $z_a = 1$ if a sensor is located on link a during departure time τ and is 0 otherwise, and $\delta_{aw}^{\tau} = 1$ if some trips with departure time τ between O-D pair w pass over link $a \in A$, and 0 otherwise. T is the planning horizon for sensor data collection.

Sensitivity Analysis on the Number of Sensors and Percentage O-D Coverage

A sensitivity analysis was conducted to explore the relationship between the number of sensors and level of O-D coverage in a network. The purpose of this analysis is to explore the marginal value, in terms of percentage coverage, of adding sensors to the network. The analysis also provides a platform to investigate the effect of sensor location on the O-D demand coverage rate.

By setting an appropriate ζ^{τ} in each departure time interval τ and solving the corresponding SLP-1 model, Figure 4 shows different sensor numbers required to provide different levels of O-D cover-

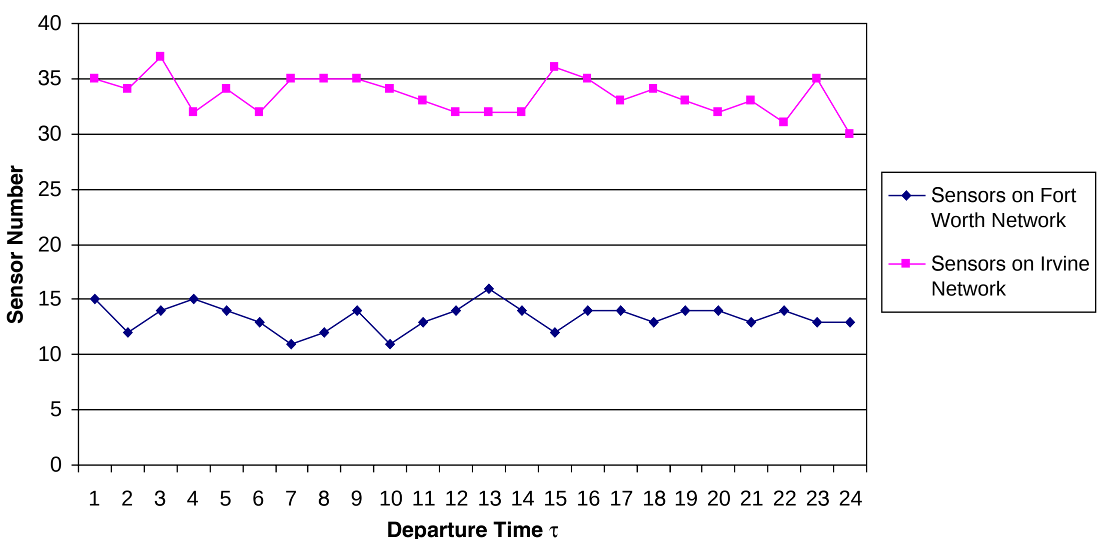


FIGURE 3 Number of sensors for each time period.

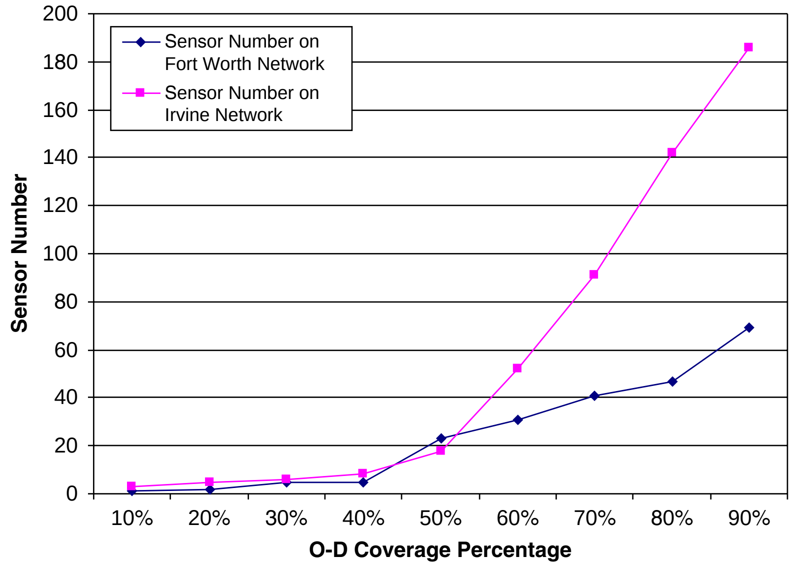


FIGURE 4 Number of sensors needed to cover given percentage of O-D demand.

age in the Fort Worth and Irvine networks under the dynamic model. As expected, to cover more O-D pairs, more sensors have to be installed in the network. These results also indicate that obtaining greater than 50% O-D coverage for either network requires a significant increase in the number of sensors. The results also indicate that a fairly low number of judiciously placed sensors can provide substantial coverage.

Figure 5 shows 23 sensors covering 50% of the O-D demand flow on the Fort Worth testbed network and 52 sensors covering 60% of the O-D demand flow on the Irvine testbed network. The sensors are mostly distributed along freeways, where the links have higher flows than on arterial streets. The results reveal that, if resources are constrained, deploying sensors along the freeway would make sense in terms of maximizing the O-D demand coverage.

Limited Network Sensors (Constrained)

This section examines the sensor location O-D coverage problem when a finite number of sensors are deployed. The solutions for the unlimited (unconstrained) sensor case that are presented in Figures 1 and 2 show a large number of sensors located on arterials and a much smaller number on the freeways. The sensitivity analysis in the preceding subsection showed that a few well-placed sensors on freeways could provide a high percentage of the O-D coverage. Because freeways tend to have higher link flows than arterials, it makes sense that a larger percentage of O-D pairs could be covered by a smaller number of links. If the number of sensors that can be placed in the network is limited, the goal becomes one of both covering the O-D pairs and intercepting as many O-D flows in the network as possible.

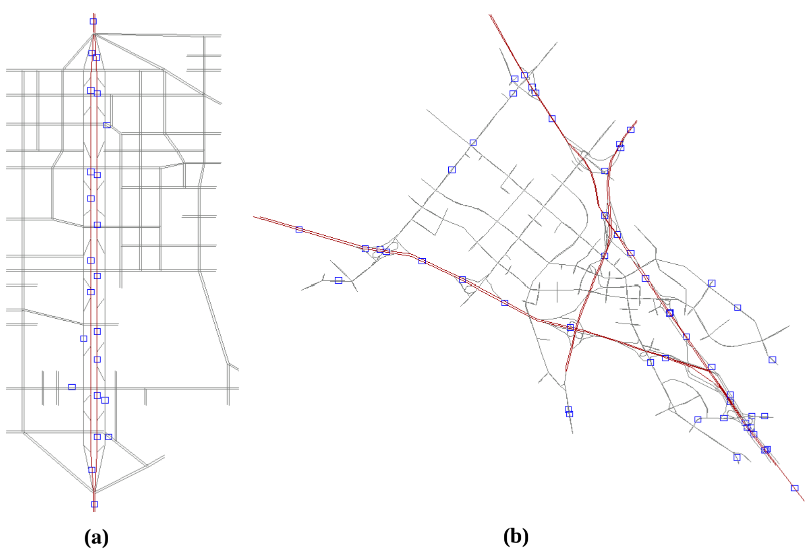


FIGURE 5 Partial O-D demand coverage on (*a*) Fort Worth and (*b*) Irvine networks.

Notations and Problem Definition

- N = set of zones, consisting of n zones;
- I = set of origin zones, consisting of n zones;
- J = set of destination zones, consisting of n zones;
- A = set of links, consisting of a links;
- W = set of O-D pairs;
- N_{od} = number of O-D pairs, $N_{od} = |I| \times |J|$;
- L = set of links with measurements;
- a = subscript for link in network, $a \in A$;
- w = subscript for O-D pair in network, $w \in W$;
- i = subscript for origin zone in network, $i \in I$;
- j = subscript for destination zone in network, $j \in J$;
- C = vector of measurements ($L \times 1$);
- H = mapping matrix ($L \times N_{od}$), mapping demand flow to link counts;
- D = demand vector, consisting of N_{od} entries $d(i, j) \in D$;
- $\hat{D}(-)$ = a priori estimated demand vector, consisting of N_{od} entries $\hat{d}_{(i,j)}(-) \in \hat{D}(-)$;
- $\hat{D}(+)$ = a posteriori estimated demand vector, $\hat{d}_{(i,j)}(+) \in \hat{D}(+)$;
- $\tilde{D}(+)$ = a posteriori estimated demand error matrix;
- $\tilde{D}(-)$ = a priori estimated demand error matrix;
- $P_{\hat{D}}(-)$ = a priori variance covariance matrix of demand matrix;
- $P_{\hat{D}}(+)$ = a posteriori variance covariance matrix of the demand matrix; and
- ϵ = vector of random noise quantities ~ $N(0, R)$ corrupting the measurements.

Generalized Least-Squares O-D Demand Estimator

Assume the relationship between the unknown O-D demand flow and measurements can be expressed as a linear combination with a random, additive measurement error ϵ. The measurement process is modeled as follows:

$$C = HD + \epsilon \tag{1}$$

The objective is to minimize the deviation between observed link flows and estimated link flows, according to the GLS estimation,

$$J = \left(C - H\hat{D}(-)\right)^T R^{-1}\left(C - H\hat{D}(-)\right) \tag{2}$$

With

$$\frac{\partial J}{\partial \hat{D}(-)} = 0$$

the resultant GLS estimator is

$$\hat{D}(-) = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} C \tag{3}$$

Assuming the measurement errors are uncorrelated (e.g., $R = I$), it is easy to prove that

$$\hat{D}(-) = \left(H^T H\right)^{-1} H^T C \tag{4}$$

For any matrix H, the rank(H) = rank(H^TH) = rank(HH^T), so that if matrix H is of full rank, then the least-squares solution $\hat{D}(-)$ is unique and minimizes the sum-of-squared residuals. In other words, the link counts on each observed link need to be linearly independent of each other.

According to Aitken's theorem (*22*), the GLS estimator $\hat{D}(-)$ is the minimum variance linear unbiased estimator in the generalized regression model.

With time-varying weighting matrices K and K', the recursive form can be expressed as

$$\hat{D}(+) = K'\hat{D}(-) + KC \tag{5}$$

because

$$\hat{D}(+) = D + \tilde{D}(+)$$
$$\hat{D}(-) = D + \tilde{D}(-) \tag{6}$$

Substituting Equations 1 and 5 into Equation 6 gives

$$\begin{aligned}\tilde{D}(+) &= K'\left(D + \tilde{D}(-)\right) + K\left(HD + \epsilon\right) - D \\ &= \left(K' + KH - I\right)D + K'\tilde{D}(-) + K\epsilon\end{aligned} \tag{7}$$

$\hat{D}(-)$ or $\hat{D}(+)$ is unbiased. That is

$$\left.\begin{aligned}E\left(\hat{D}(+)\right) = E\left(D + \tilde{D}(+)\right) = D + E\left(\tilde{D}(+)\right) \\ E\left(\hat{D}(-)\right) = E\left(D + \tilde{D}(-)\right) = D + E\left(\tilde{D}(-)\right)\end{aligned}\right\} \Rightarrow \begin{cases}E\left(\tilde{D}(+)\right) = 0 \\ E\left(\tilde{D}(-)\right) = 0\end{cases} \tag{8}$$

By definition, $E(\epsilon) = 0$, Equations 7 and 8 give

$$K' = \left(I - KH\right) \tag{9}$$

Substituting Equation 9 into Equation 7

$$\tilde{D}(+) = \left(I - KH\right)\tilde{D}(-) + K\epsilon \tag{10}$$

By definition, the posterior error variance covariance matrix

$$\begin{aligned}P_{\hat{D}}(+) &= E\left(\tilde{D}(+)\right) - E\left(\tilde{D}(+)\right)\left(\tilde{D}(+)\right) - E\left(\tilde{D}(+)\right)^T \\ &= E\left(\tilde{D}(+)\tilde{D}(+)^T\right)\end{aligned} \tag{11}$$

Substituting Equation 10 into Equation 11,

$$\begin{aligned}P_{\hat{D}}(+) &= \left(\left(I - KH\right)\tilde{D}(-) + K\epsilon\right)\left(\left(I - KH\right)\tilde{D}(-) + K\epsilon\right)^T \\ &= \left(I - KH\right)P_{\hat{D}}(-)\left(I - KH\right)^T + KRK^T\end{aligned} \tag{12}$$

To minimize $P_{\hat{D}}(+)$, the first-order optimization condition needs to be satisfied,

$$\frac{\partial P_{\hat{D}}(+)}{\partial K} = -2\left(I - KH\right)P_{\hat{D}}(-)H^T + 2KR = 0 \tag{13}$$

Thus, the optimal weight matrix, which is referred to as the Kalman gain matrix is

$$K = P_{\hat{D}}(-)H^T\left(HP_{\hat{D}}(-)H^T + R\right)^{-1} \tag{14}$$

Substituting Equation 14 into Equation 12, the minimal updated variance covariance matrix is

$$P_{\hat{D}}(+) = (I - KH)P_{\hat{D}}(-) \tag{15}$$

A simple form of Kalman gain matrix can be expressed as

$$K = P_{\hat{D}}(+)H^T R^{-1} \tag{16}$$

Equation 12 can be also expressed as

$$P_{\hat{D}}^{-1}(+) = P_{\hat{D}}^{-1}(-) + H^T R^{-1} H \tag{17}$$

More detailed derivations and analysis of the optimal estimation and filtering relationship can be found elsewhere (*24*).

If it is assumed that the measurement error is independent, then R is a diagonal matrix. So, Equation 16 can be written as follows:

$$K = \frac{P_{\hat{D}}(+)H^T}{R} \tag{18}$$

The matrix H is a mapping matrix, mapping the O-D demand flow to the link counts; if it is assumed to be an identity matrix, one gets

$$K = \frac{P_{\hat{D}}(+)}{R} \tag{19}$$

The Kalman gain matrix in the sensor location problem can be interpreted as the summation of information gain contributed by each O-D pair passing over that observation link. It is “proportional” to the uncertainty in the estimate and “inversely proportional” to the measurement noise (*24*). The relationship of Equation 15 declares that, given the a priori variance covariance estimated demand error, the larger the gain a link has, the more information it can collect to correct the estimated error. Compared with the a posteriori variance covariance matrix, the gain matrix is much more sensitive to the measurement errors that influence the estimated results.

On the basis of the preceding analysis, the objective in the constrained sensor coverage model is to find the set of links that can maximize the total information gains, constrained by the link independence and resource constraints. To maintain the unbiasedness of the O-D flow estimator, link independence should be satisfied.

To find the mapping matrix, H or the so-called time-dependent link flow proportion matrix, the a priori estimated traffic demand should be assigned to the network according to some assignment rules (*25*). In this study, the drivers in the network were assumed to take the paths consistent with those generated under the dynamic user equilibrium assignment. Dynamic user equilibrium and system optimization procedures are important components of Dynasmart-P (*22*), which is used to solve the dynamic user equilibrium assignment problem and find corresponding simulated time-dependent link flows and mapping matrix H in this study.

Sensor Location Model and Algorithm

If $L \geq \hat{L}_0$, where $\hat{L}_0$ is an optimal solution to SLP-1, the sensor locations could cover the relevant subset of O-D pairs. The problem can be formulated as SLP-2.

$$\text{SLP-2} \quad \text{maximize} \sum_{a \in A} K_a z_a \tag{SLP-2a}$$

subject to

$$\sum_{a \in A} z_a \leq L \qquad w \in W \tag{SLP-2b}$$

$$\sum_{a \in A} \delta_{aw}^{\tau} z_a \geq 1 \qquad w \in W,\ \tau \in T \tag{SLP-2c}$$

$$K_a = \sum_{w \in W} \frac{P_{\hat{D}}(-)H_a^T}{H_a P_{\hat{D}}(-)H_a^T + r_a},\ a \in A \tag{SLP-2d}$$

$$\delta_{aw}^{\tau} = \text{assignment } \lfloor d_w \rfloor \text{ from DTA}, \quad a \in A,\ w \in W,\ \tau \in T \tag{SLP-2e}$$

$$z_a = 0, 1,\ a \in A \tag{SLP-2f}$$

If $L < \hat{L}_0$, only partial relevant O-D pairs can be covered; thus, the problem is formulated as SLP-2′.

$$\text{SLP-2}' \quad \text{maximize} \sum_{a \in A} K_a z_a$$

subject to

$$\sum_{a \in A} z_a \leq L \qquad w \in W$$

$$K_a = \sum_{w \in W} \frac{P_{\hat{D}}(-)H_a^T}{H_a P_{\hat{D}}(-)H_a^T + r_a},\ a \in A$$

$$z_a = 0, 1,\ a \in A$$

Considering the magnitude of the O-D demand flow relative to the variances, the model can be formulated as follows:

If $L \geq \hat{L}_0$,

$$\text{SLP-3} \quad \text{maximize} \quad \text{trace}\left(\frac{E(\hat{D}(+))}{P_{\hat{D}}(+)}\right)$$

subject to

$$\sum_{a \in A} z_a \leq L \qquad w \in W$$

$$\sum_{a \in A} \delta_{aw}^{\tau} z_a \geq 1 \qquad w \in W,\ \tau \in T$$

$$P_{\hat{D}}^{-1}(+) = P_{\hat{D}}^{-1}(-) + \frac{H^T H}{R}$$

$$\hat{D}(+) = D + \tilde{D}(+)$$

$$z_a = 0, 1,\ a \in A$$

$$\delta_{aw}^{\tau} = \text{assignment } \lfloor d_w \rfloor \text{ from DTA},\ a \in A,\ w \in W,\ \tau \in T$$

If $L < \hat{L}_0$

$$\text{SLP-3}' \quad \text{maximize} \quad \text{trace}\left(\frac{E(\hat{D}(+))}{P_{\hat{D}}(+)}\right)$$

subject to

$$\sum_{a \in A} z_a \leq L \qquad w \in W$$

$$P_{\hat{D}}^{-1}(+) = P_{\hat{D}}^{-1}(-) + \frac{H^T H}{R}$$

$$\hat{D}(+) = D + \tilde{D}(+)$$

$$z_a = 0, 1,\ a \in A$$

The proposed models are computationally intensive. The major difficulty is associated with calculating the Kalman gain matrix, because matrix inversion occurs at each time interval. The computational intensity is especially noticeable in a large-scale network. The sequential algorithm by Chui and Chen (*26*) was designed to avoid direct computation of the inversion of the matrix, $HP_{\hat{D}}(-)H^T + R$ by assuming independence of the link measurement errors. Figure 6 illustrates the sequential algorithm for the sensor location problem [similar to the sequential algorithm of Chui and Chen (*26*)].

BnB is a common strategy used to solve integer programs. BnB algorithms have been developed in a variety of areas. Because of its adaptability, BnB has been used in a variety of search algorithms, such as best-first search and depth-first search, as well as others.

To accommodate solving the sensor location problem in a large-scale network, Algorithm 1 integrates the BnB algorithm with the sequential algorithm. Through the use of an efficient search algorithm, Algorithm 1 can be used to solve the SLP in a large-scale network. However, as the size of the network grows, the efficiency of analyzing different sensor location strategies will be reduced.

Algorithm 1 [Sequential Algorithm + BnB Algorithm]

Step 0 (initialization). Running DTA simulation software (*22*) given a prior O-D demand matrix to get link flow proportion H_a under user equilibrium assignment, $a \in A$. Given $P_{\hat{D}}(-) = \hat{P}_0$, which can be from historical O-D data statistics or traffic-planning agents.

Step 1 (gain calculation). Using the sequential algorithm calculating the link gains across the network.

Step 2 (BnB algorithm). With the calculated link gains across the network, solving the proposed model as a binary integer model using the BnB algorithm.

An intuitive notion to solve the proposed model is selecting L links every time from the network $G(V, A)$, calculating the total link gains each time and selecting the locations having the largest link

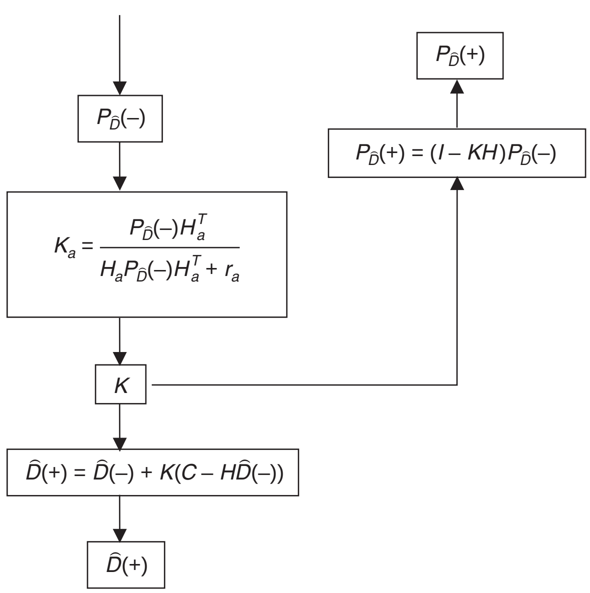


FIGURE 6 Sensor location problem sequential algorithm flowchart.

gains. However, the number of combinations of L links from a total of A links is

$$\binom{A}{L} = \frac{A!}{L!(A-L)!}$$

and the complexity of the computation will become nonpolynomial. Based on this notion, Heuristic 2 is developed to find the best feasible solution.

Heuristic 2

Step 0 (initialization). Run DTA simulation software (*22*) given a priori O-D demand matrix to obtain link flow proportions H_a under user equilibrium assignment, $a \in A$. Calculate and sort the information gain on each link of the network using the previously mentioned sequential algorithm. Given $P_{\hat{D}}(-) = \hat{P}_0$, which could be obtained from historical O-D data statistics or traffic-planning agencies. Set $LS = \phi$, $\overline{LS} = A - LS$, $S_{od} = \phi$, $\overline{S}_{od} = W$, where S_{od} is a set of O-D pairs covered by sensors in set LS. $N_{sensor} = 0$, where N_{sensor} is the total number of sensors in set LS.

Step 1 (stopping criterion). If number of links (N_{sensor}) in set LS, $N_{sensor} = L$, where L is a given sensor number or $\overline{S}_{od} = \phi$ or the preset computation time is reached, stop. Output the best feasible solution in set LS, Otherwise, go to Step 2.

Step 2 (maximal gain selection). If $LS = \phi$, select link l_i, so that $\text{gain}_i \geq \text{gain}_j$, $\forall j$, $i \neq j$, $i, j \in \overline{LS}$. Insert the selected link l_i into link set LS and let $N_{sensor} = N_{sensor} + 1$, $S_{od} = \{N_{l_i}\}$, $\overline{S}_{od} = W - S_{od}$, where N_{l_i} is the set of O-D pairs newly covered by the selected link l_i, delete the selected link l_i from set $\overline{LS}$, go to Step 1. Otherwise, if $LS \neq \phi$, go to Step 3.

Step 3 (link selection). For each link in link set $\overline{LS}$, the O-D pairs covered by the link and in set S_{od} are marked by a + and the O-D pairs covered by l_i and in set $\overline{S}_{od}$ are marked by a –. $n_1(i)$ is the number of + values and $n_2(i)$ is the number of – values. Select link l_i so that $n_1(i) \geq n_1(j)$, $\forall j$, $i, j \in \overline{LS}$.

If there exist links l_i, l_j so that $n_1(i) > 0$, $n_1(j) > 0$, and $n_2(i) > n_2(j) = 0$, $i, j \in \overline{LS}$, select link l_j.
Else if $n_1(i) > 0$, $n_1(j) > 0$ and $n_2(i) = n_2(j) = 0$, $i, j \in \overline{LS}$.
  If link gain on link $i >$ link gain on link j, select link l_i.
  Else if link gain on link $i \leq$ link gain on link j, select link l_j.
Else if there exist links l_i, l_j so that $n_1(i) > n_1(j) > 0$, $i, j \in \overline{LS}$, select link l_i.
Else if there exist links l_i, l_j so that $n_l(i) = n_l(j) > 0$, $i, j \in \overline{LS}$.
  If $n_2(i) > n_2(j)$, select link l_j.
  Else if $n_2(i) < n_2(j)$, select link l_i.
  Else if $n_2(i) = n_2(j) \neq 0$, select the link with less measurement error.
  Else if $n_2(i) = n_2(j) = 0$, select the link with larger link information gain.
End if

Moved the O-D pairs covered by the selected link from set $\overline{S}_{od}$ to set S_{od}. $N_{sensor} = N_{sensor} + 1$. Go to Step 1.

As mentioned in last section, the link counts on each observed link need to be linearly independent of each other. The basic idea in the proposed heuristic is to select links with the largest information gain while keeping the rank of link proportion matrix H full. Thus, the complexity of the proposed heuristic is determined by the complexity of finding maximal O-D coverage given the simulated link flow proportions, H_a, $a \in A$ from Dynasmart-P and sorted link information gains. In general, this problem can be classified as a maximum rank matrix completion problem, which assigns values from some set to maximize the rank of the matrix. It can be shown that the computational complexity of the proposed algorithm is $O(n^3)$, where n is the number of links in the network.

NUMERICAL EXAMPLES

A series of examples based on a six-node network is used to demonstrate the proposed methodology. To facilitate the ability to compare the results of this research with the recent results of Zhou and List (*21*), the same example network was used.

The first example is a single-point sensor location, according to the setup in Figure 7. O-D Pair 1 is from Node 1 to Node 2 and O-D Pair 2 is from Node 1 to Node 3; O-D Pair 1 has two routes; 70% of the flow travels along path {1 4 5 2} and the remaining 30% of the flow travels along path {1 4 6 5 2}. Both O-D pairs have a flow volume of 20 units. Assume

$$P_{\hat{D}}(-) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$

meaning that O-D Pair 1 has a larger a priori variance than O-D Pair 2. The standard deviation of the measurement error for a sensor is assumed to be 5% of the corresponding true flow volume.

The sensor in Figure 7*b* covers O-D Pair 1 with larger variance producing larger gain than that in Figure 7*c*. Because the sensor in Figure 7*d* covers both O-D pairs and intercepts the largest O-D flows in these scenarios, it collects the largest gain through the observation counts even though the sensor in Figure 7*d* brings a larger error than that in Figure 7*b* and *c*. If the error in Figure 7*d* is reduced to 1, similar to that in Figure 7*b* and *c*, it has $R = 1$, $K = [0.6667\ 0.1667]^T$, and gain = 8.337, producing larger information gain.

Figure 8 presents examples of single sensor locations with route choice. Figure 8*a* presents an error-free link proportion estimate and the measurement error proportional to the link flow scenario. The gain in that scenario is 1.1429, which is greater than all the scenarios in Figure 6. This indicates that the measurement error could result in reduction of the link information gain. Figure 8*b* considers the link proportion estimation error, which reduces the information gains. Although the sensor in Figure 8*c* covers both O-D pairs, it still cannot produce the largest information gain because of the largest measurement error in the three scenarios. Even when the measurement error is reduced to 1, the gain matrix is $K = [0.5085\ 0.4237]^T$ and gain = 0.9322.

Figure 9 presents examples of two sensor locations. Figure 9*a* covers O-D Pair 1, Figure 9*b* covers O-D Pair 2, Figures 9*c* and *d* cover both O-D pairs, and Figure 9*e* covers O-D Pair 1 but the two sensors have a measurement error correlation between them. As expected, Figure 9*c* collected larger gains than the other scenarios because it covers both O-D pairs. Although Figure 9*d* covers both O-D pairs as well, the information gain is smaller than in Figure 9*c*

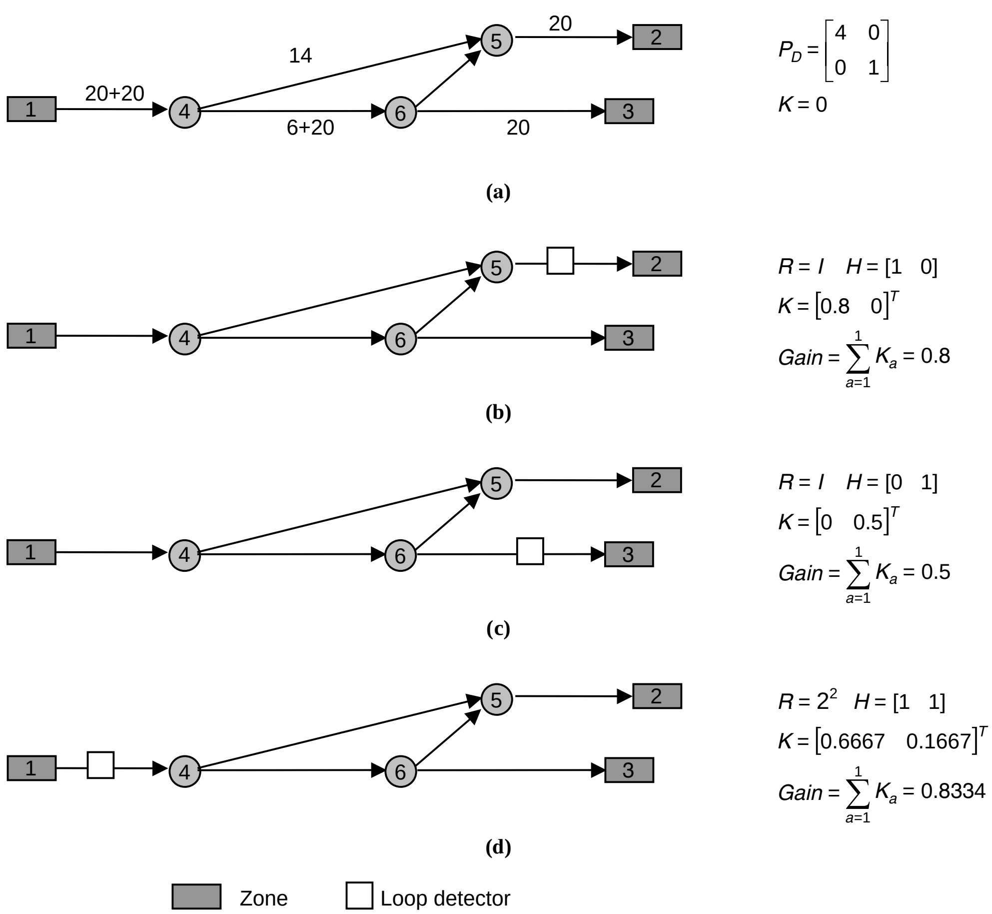


FIGURE 7 Single-point sensor locations: (*a*) base case, (*b*) one sensor for O-D pair (1–2), (*c*) one sensor for O-D pair (1–3), and (*d*) one sensor for both O-D pairs.

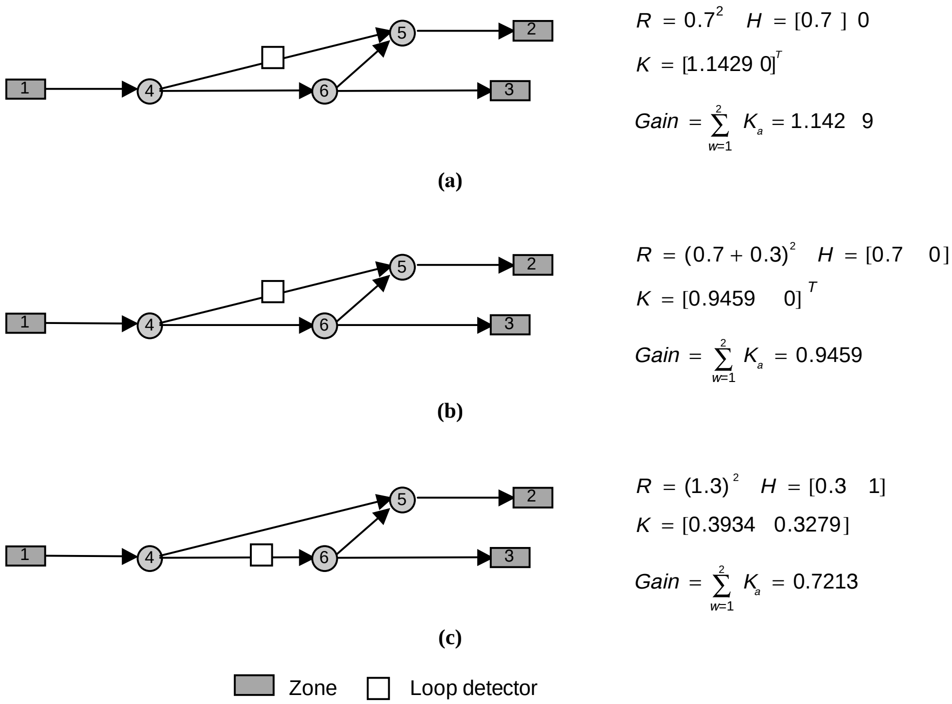


FIGURE 8 Single-point sensor locations with route choice: (*a*) assignment error-free with link proportion 0.7, (*b*) standard of link proportion estimation errors (from traffic assignment) = 0.3, and (*c*) assignment error-free with link proportion of 0.3.

because of the linear dependence of the two observations. Comparing Figure 9*a* and *e*, the correlation of measurement errors provided some reduction in the information gain.

Figure 10 presents examples of three sensor locations. Figure 10*e* collected the least information gain because the three sensors cover only one O-D pair, whereas the other scenarios cover both O-D pairs. Figure 10*a* produces the best gain because of the link independent of the sensor data.

More sensors do not always result in more information gain. Figure 9*c* with two sensors (gain = 1.3) has larger gains than most scenarios in Figure 10. Even if the two cases have the same measurement errors, in Figure 10*e* one O-D pair has $K = \begin{bmatrix} 0.3077 & 0.3077 & 0.3077 \\ 0 & 0 & 0 \end{bmatrix}$, gain = 0.9231, which is less than that in Figure 9*c*.

This example is also used to demonstrate the procedure of Algorithm 2 for finding the best feasible solution of sensor locations.

Step 0 (initialization).

$$R = 2^2 \quad H = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Link 1. $K = \begin{bmatrix} 0.6667 & 0.1667 \end{bmatrix}^T$

$$\text{gain} = \sum_{w=1}^{2} K_a = 0.8334$$

$$R = 0.7^2 \quad H = \begin{bmatrix} 0.7 & 0 \end{bmatrix}$$

Link 2. $K = \begin{bmatrix} 1.1429 & 0 \end{bmatrix}^T$

$$\text{gain} = \sum_{w=1}^{2} K_a = 1.1429$$

$$R = (1.3)^2 \quad H = \begin{bmatrix} 0.3 & 1 \end{bmatrix}$$

Link 3. $K = \begin{bmatrix} 0.3934 & 0.3279 \end{bmatrix}$

$$\text{gain} = \sum_{w=1}^{2} K_a = 0.7213$$

$$R = 1 \quad H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Link 4. $K = \begin{bmatrix} 0.8 & 0 \end{bmatrix}^T$

$$\text{gain} = \sum_{w=1}^{2} K_a = 0.8$$

$$R = 1 \quad H = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Link 5. $K = \begin{bmatrix} 0 & 0.5 \end{bmatrix}^T$ $\qquad P_{\hat{D}}(-) = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

$$\text{gain} = \sum_{w=1}^{2} K_a = 0.5$$

$$LS = \phi, \overline{LS} = \{1,2,3,4,5\}, S_{\text{od}} = \phi, \overline{S}_{\text{od}} = \{1,2\}, N_{\text{sensor}} = 0, H = \text{NULL}$$

1. One sensor in the network. First, select Link 2, which has maximal gain over all five links, $H = [0.7 \;\; 0]$, $\text{rank}(H) = 0 + 1 = 1$, so $LS = \{2\}$, $\overline{LS} = \{1,3,4,5\}$, $S_{\text{od}} = \{1\}$, $N_{\text{sensor}} = 1$, then go to Step 1, the number of selected links equals given sensor number, stop. Comparing the sensor on Link 1 that covers both O-D pairs, the sensor on Link 2 covers only O-D Pair 1. However, the root-mean-squared error (RMSE) of the sensor on Link 2 is 1.3416, which is less than 1.7638, the RMSE of the sensor on Link 1. Thus, the sensor on Link 2 can provide a more reliable O-D matrix than the estimated *y* sensor data on Link 1.

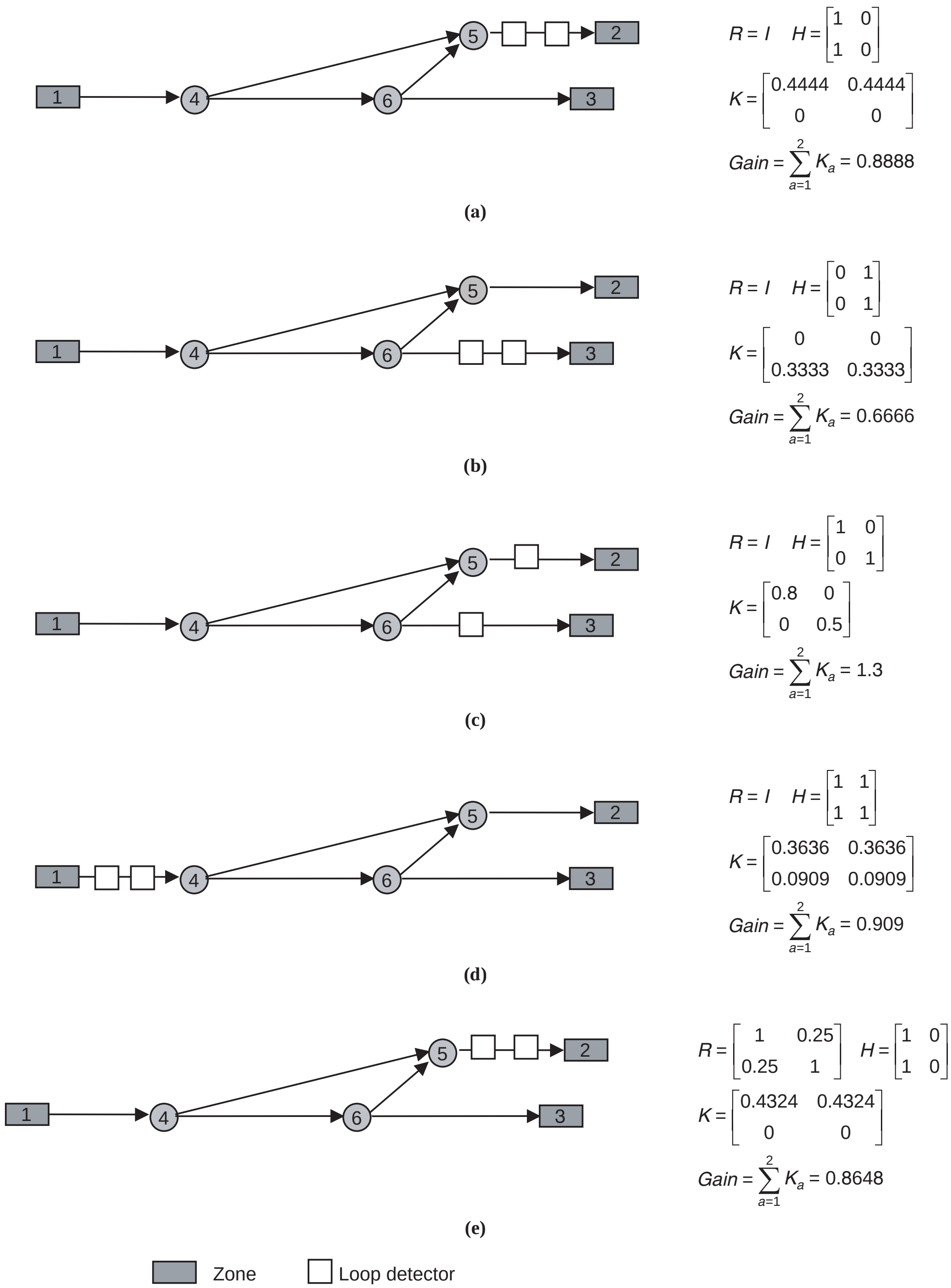
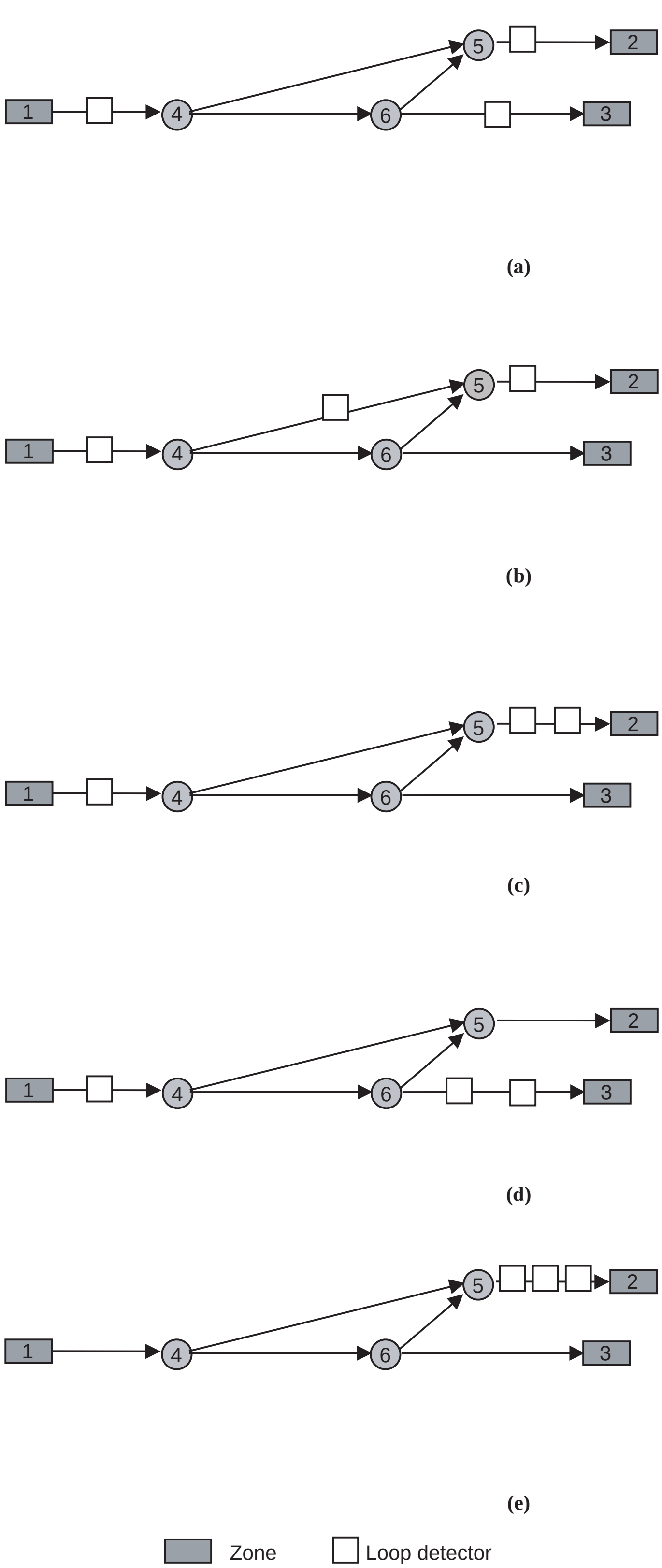


FIGURE 9 Two-point sensor locations: (*a*) two uncorrelated sensors for O-D pair (1–2), (*b*) two uncorrelated sensors for O-D pair (1–3), (*c*) two uncorrelated sensors for both O-D pairs, (*d*) two uncorrelated sensors for both O-D pairs, and (*e*) two partially correlated sensors for O-D pair (1–2).



$$R = 1.5I \qquad H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.3419 & 0.4786 & -0.1368 \\ 0.1880 & -0.1368 & 0.3248 \end{bmatrix}$$

$$Gain = \sum_{a=1}^{3} K_a = 1.3333$$

(a)

$$R = 1.5I \qquad H = \begin{bmatrix} 1 & 1 \\ 0.7 & 0 \\ 1 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.2434 & 0.2840 & 0.4057 \\ 0.3026 & -0.1136 & -0.1623 \end{bmatrix}$$

$$Gain = \sum_{a=1}^{3} K_a = 1.2357$$

(b)

$$R = 1.5I \qquad H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.2017 & 0.3361 & 0.3361 \\ 0.3193 & -0.1345 & -0.1345 \end{bmatrix}$$

$$Gain = \sum_{a=1}^{3} K_a = 1.1932$$

(c)

$$R = 1.5I \qquad H = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.6747 & -0.1928 & -0.1928 \\ 0.0723 & 0.2651 & 0.2651 \end{bmatrix}$$

$$Gain = \sum_{a=1}^{3} K_a = 1.2772$$

(d)

$$R = 1.5I \qquad H = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.2963 & 0.2963 & 0.2963 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Gain = \sum_{a=1}^{3} K_a = 0.8889$$

(e)

FIGURE 10 Three-point sensor locations: (*a*) three uncorrelated sensors for both O-D pairs, (*b*) three uncorrelated sensors for both O-D pairs, (*c*) three uncorrelated sensors for both O-D pairs, (*d*) three uncorrelated sensors for both O-D pairs, and (*e*) three uncorrelated sensors for O-D pair (1–2).

2. Two sensors in the network. After finding the first sensor on Link 2, go to Step 3. $n_1(1) = n_1(3) = n_1(5) = 1$, $n_2(1) = n_2(3) = 1$, $n_2(5) = 0$. Link 5 is selected, $LS = \{2,5\}$, $\overline{LS} = \{1,3,4\}$, $S_{od} = \{1,2\}$, $N_{sensor} = 2$. Go to Step 1, stop. The total gain of sensors on Links 2 and 5 is 1.6429, whereas the total gain of sensors on Links 2 and 1 is 1.2956, and the total gain of sensors on Links 2 and 3 is 1.562.

The Irvine network (Figure 1) was used to demonstrate the proposed model. The simulation experiments were implemented on an Intel Xeon central processing unit 3.20-GHz 64-bit machine with 8G memory.

The historical 2-h time-dependent O-D volumes were integrated into one demand table as an a priori mean estimate because of the limited sensor number constraint. It is assumed that the standard deviation of the a priori demand variance is 30% of the corresponding demand volume in the historical demand table. The standard deviation of the measurement error is assumed constant. Dynasmart-P (*22*) was again used to assign the O-D volumes onto the network and represent traffic flow evolution. Figure 11 shows the network covered by 10 sensors. The two O-D pairs carrying the largest O-D volume are (53, 46) and (47, 52), accounting for about 36% of the total historic O-D demand. Sensor 1 and Sensor 2 covered these two O-D pairs. In fact, as illustrated in the previous small network, the independent sensors usually gave high link information gains. It is not sufficient to minimize the variance of the estimated O-D demand for one particular O-D pair while leaving other O-D pairs uncovered. The proposed method tries to find the sensor locations that garner the largest possible link information gains in conjunction with maximizing network coverage. The 10 sensors shown in Figure 11*a* covered about 51% of the total O-D trips and the 20 sensors shown in Figure 11*b* covered about 64% of the total O-D trips.

As discussed in the first section, only if the link mapping matrix H has full rank is the O-D demand estimator $\hat{D}(-)$ the best linear unbiased estimator. The gain matrix was derived based on the best linear unbiased estimator, which explained why the independent sensor data always produced the largest gains. The following observations are made from those example results to maximize information gains:

1. The sensors need to be located on the links that can intercept the most O-D flows.
2. The sensor observation data should be linearly independent.
3. More sensors do not necessarily mean larger information gains.
4. The lower the measurement error, the greater the gain the system may attain.

ANALYSIS MEASURES

To assess the impact of different sensor location strategies in conjunction with the O-D demand estimator error reduction, the RMSE of the O-D demand will be calculated to check the quality of the estimated O-D matrix. The RMSE is simply the square root of the mean squared error (MSE).

Proposition

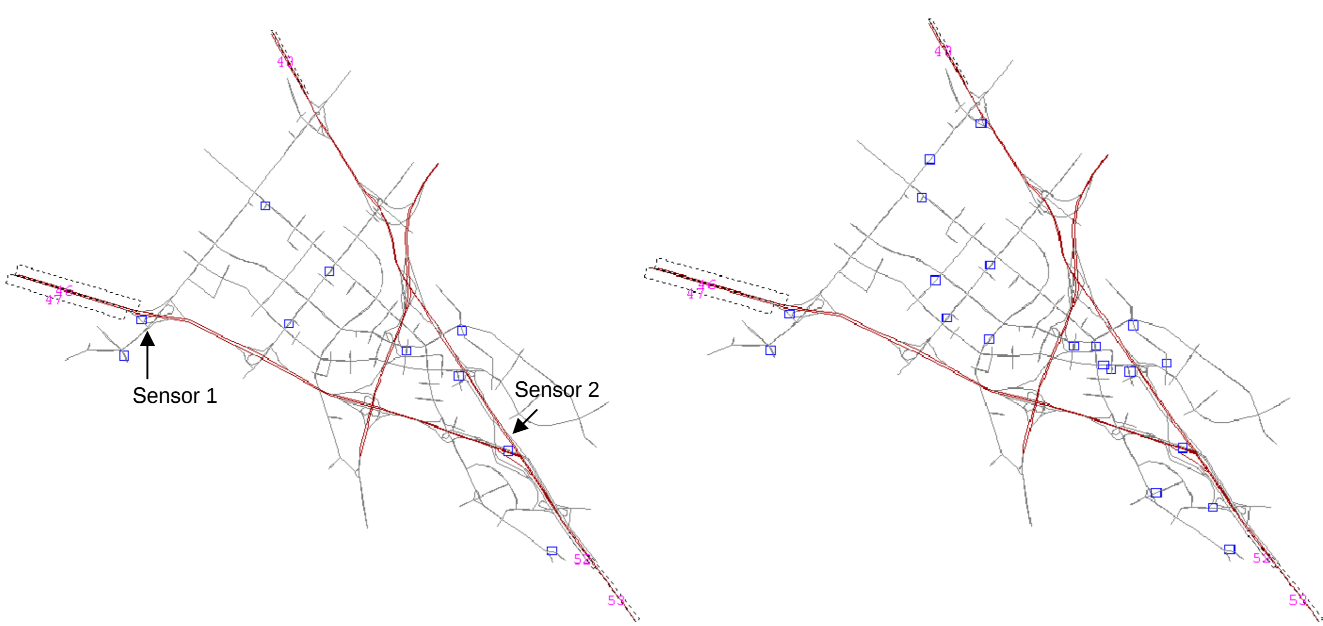
The proposed models always produce the minimal MSE across all other O-D estimators.

Proof

In statistics, the MSE is defined (*27*) as follows:

$$\mathrm{MSE}\left(\theta|\hat{\theta}\right) = \mathrm{var}\left(\hat{\theta}\right) + E\left[\left(\hat{\theta} - \theta\right)\left(\hat{\theta} - \theta\right)^T\right] \qquad (20)$$

As mentioned previously, the GLS O-D demand estimator is unbiased; thus, its MSE matrix is its covariance matrix. Recall Equation 17—the MSE of the O-D estimator is $P_{\hat{D}}^{-1}(+) = P_{\hat{D}}^{-1}(-) + H^T R^{-1} H$. Because $P_{\hat{D}}(-)$ is the a priori variance covariance matrix of the demand matrix and the objectives of both of the SLP-2 and SLP-3



(a) (b)

FIGURE 11 Sensor locations by link gain selection in Irvine network: (*a*) 10 sensors and (*b*) 20 sensors.

models are indirectly minimizing $P_{\hat{D}}(+)$, the MSE based on the proposed models is thus the minimal statistics inference across all other estimators. This completes the proof.

Under the time-dependent condition, the time dimension needs also to be considered; the calculation is as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{\tau}\sum_{w}\left(d_w^{\tau} - \hat{d}_w^{\tau}\right)^2}{|W| \times T}}$$

where

d_w^{τ} = ground truth O-D trips of O-D pair w at departure time τ,
$\hat{d}_w^{\tau}$ = estimated O-D trip of O-D pair w at departure time τ,
W = set of O-D pairs,
$|L|$ = total number of O-D pairs in the set, and
T = number of departure time intervals.

In a given sensor location strategy, the RMSE is calculated across all the O-D pairs and across all the time intervals. Table 1 shows the RMSE of the previously mentioned numerical examples.

CONCLUDING REMARKS

This paper presents models that use the Kalman filtering method to explore time-dependent maximal information gains across all the links in the network. The research proposes two types of sensor location models to solve an O-D coverage problem and a maximal information gain driven problem. The focus is on solving the sensor location problem as an O-D coverage problem under a DTA. A sensitivity analysis is conducted to explore the relationship between the number of sensors and the level of O-D coverage in a network. The goal is to produce a quality estimated O-D matrix that integrates link observation data that minimize the variance of the O-D flow estimator. This research constructed an unbiased generalized least-squares estimator, using a linear relationship and link flow proportions obtained from a dynamic simulation-assignment procedure (Dynasmart-P). The models were developed to identify link sensor locations that produce maximal information gains and maximal O-D pair reliability. In addition, a sequential algorithm was developed to solve the proposed models. Several small numerical examples were used to demonstrate the proposed methodology. Finally, the detector configuration was evaluated on the basis of RMSE to prove the efficiency of the proposed methodology.

Recognizing the importance of sensor location and its relationship to the quality of an estimated O-D matrix, this paper established a connection between the two critical issues of traffic sensor location and estimation error based on Kalman filtering. Ongoing research is exploring efficient algorithms that can be used to solve the sensor location problem for O-D estimation in a real-time setting, using real-time observation data. In addition, an efficient framework is being designed to embed the proposed methodology into a simulation-based real-time network traffic estimation and prediction system (Dynasmart-X) that relies on DTA methodology to take full advantage of the dynamic sensor locations to estimate and predict O-D matrices in large-scale networks.

TABLE 1 RMSE for Numerical Examples

Scenario	(a)	(b)	(c)		
One sensor coverage without route choice					
RMSE	1.3416	2.1213	1.7638		
One sensor coverage with route choice					
RMSE	1.1986	1.3416	1.5681		
Scenario	(a)	(b)	(c)	(d)	(e)
Two sensor coverage					
RMSE	1.2019	2.0817	1.1402	1.3817	1.2412
Three sensor coverage					
RMSE	1.0978	1.1428	1.0885	1.3034	1.2019

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