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Transportation Research Part C 16 (2008) 371–389

TRANSPORTATION RESEARCH PART C

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A bi-criterion dynamic user equilibrium traffic assignment model and solution algorithm for evaluating dynamic road pricing strategies

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Received 20 February 2007; received in revised form 19 August 2007; accepted 20 August 2007

Abstract

To support the evaluation of dynamic road pricing strategies in a network context, this study develops a bi-criterion dynamic user equilibrium (BDUE) model, which aims to capture users' path choices in response to time-varying toll charges, and hence explicitly considers heterogeneous users with different value of time (VOT) preferences in the underlying path choice decision framework. The VOT is represented as a continuously distributed random variable across the population of trips, and an infinite dimensional variational inequality formulation is established to characterize the solutions to the BDUE problem. This study proposes a simulation-based heuristic approach to find the BDUE path flow patterns. Specifically, embedded in the algorithmic framework are (i) a bi-criterion time-dependent least generalized cost path algorithm applied to generate the extreme efficient path set and the corresponding set of breakpoints that naturally defines the multiple user classes, thereby generating the descent direction from a multi-class auxiliary path flow vector, and (ii) a traffic simulator used to describe the traffic flow propagation and determine experienced travel times and costs. To circumvent the difficulty of storing the memory-intensive path set and routing policies for large-scale network applications, a vehicle-based implementation technique is proposed to use the vehicle path set as a proxy for keeping track of the path assignment results. A set of numerical experiments are conducted on several real road networks to explore the convergence behavior and solution quality of the BDUE algorithm, as well as to investigate how VOT distributions affect the path flow pattern and toll road usage under dynamic road pricing scenarios.

Keywords: Dynamic traffic assignment; Bi-criterion dynamic user equilibrium; Dynamic road pricing; Value of time; Heterogeneous driver preferences; Network models

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0968-090X/$ - see front matter

doi:10.1016/j.trc.2007.08.002

1. Introduction

Road (congestion) pricing has long been considered by economists (since William Vickrey, the Nobel Prize winner in Economics in 1952, first proposed the concept and principles; Vickrey, 1969) and transportation authorities as an effective demand management strategy to reduce traffic congestion and improve system performance during peak periods in many metropolitan areas, notwithstanding a general attitude of public opposition. Because of the time-varying nature of congestion, dynamic road pricing has recently drawn increasing attention from the congestion pricing research community (Arnott et al., 1990; Wie and Tobin, 1998; Joksimovic et al., 2005). To support the planning, operation, and evaluation of various dynamic road pricing schemes, a user equilibrium dynamic traffic assignment (DTA) model is often applied to predict path choices and the resulting network flow patterns, which in turn form the basis for assessing the economic and financial impacts or benefits of proposed toll facilities or schemes. To this end, a path choice model that can realistically capture trip-makers' decisions in response to toll charges is critical to the development of DTA models for analyzing dynamic road pricing scenarios. Therefore, this study aims at developing a bi-criterion dynamic user equilibrium (BDUE) traffic assignment model which explicitly considers heterogeneous users that seek to minimize the *two* essential *criteria*, travel time and out-of-pocket cost, in the underlying path choice framework. The term ‘‘bi-criterion’’ was also suggested by Dial (1996, 1997) for the static UE assignment problem with the value of time (VOT) distributed across users.

Generally, in the literature, two types of path choice models are employed in network equilibrium assignment models for road pricing applications. The first type is characterized by probabilistic discrete path choice (e.g. logit- or probit-based) models consisting mainly of the aforementioned two path travel attributes. Those discrete choice models can be constructed and calibrated from revealed or stated preference survey analyses to determine the probability that a trip-maker will use paths that include tolled facilities (e.g. Nielsen et al., 2002; Florian et al., 2006; Dial, 1979; Cantarella and Binetti, 1998). The other type implements deterministic path choice models based on the generalized path cost (time) function in which path travel time (path cost) is weighted by a trip-maker's VOT representing how much money the trip-maker is willing to tradeoff for unit time saving (e.g. Leurent, 1993; Dial, 1996).

Conventional (static) traffic assignment models (e.g. Yang and Meng, 2000) for road pricing applications assume homogeneous perception of tolls for all trip-makers; that is, every trip-maker is willing to trade off the same amount of money for a unit time saving, corresponding to the constant coefficients associated with the path travel time and travel cost in the generalized path cost function. However, empirical studies (e.g. Ben-Akiva et al., 1993; Hensher, 2001) have found that path choice models with random coefficients have better goodness of fit than those with constant coefficients. Others (e.g. Small and Yan, 2001; Brownstone and Small, 2005; Small et al., 2005; Cirillo and Axhausen, 2006) suggested that the VOT varies significantly across individuals because of different socio-economic characteristics, trip purposes, attitudes and inherent preferences. This user heterogeneity is manifested in the fact that some trips take slower paths to avoid tolls while others choose toll roads to save time. Therefore, it is essential to explicitly recognize and represent heterogeneous users in modeling users' response to toll charges in DTA models for road pricing applications. This is especially important in assessing the feasibility of a proposed toll facility and its financial viability from the standpoint of the public or private entity who will be operating it.

The BDUE model proposed in this study addresses this fundamental question by considering a path choice model based on the generalized path cost function approach. The behavioral assumption of this approach is that, in a disutility-minimization framework, each trip will use the least generalized cost path among the competing paths, with the generalized cost being the sum of path cost and path time weighted by the trip-maker's VOT, which is considered as a random variable distributed across the population of trips. When embedded in network equilibrium assignment models for road pricing applications, this path choice model approach typically involves solving the bi-criterion shortest path problem that determines a set of non-dominated (or efficient) paths, because no unique optimal path usually exists in terms of all the objectives (or simultaneously minimizes conflicting attributes). For a review of bi-criterion or multi-criterion shortest path algorithms, the reader is referred to Nielsen (2003) and Mahmassani et al. (2005b).

In the static traffic assignment context, previous studies that address user heterogeneity and are based on the generalized path cost function approach can be classified into two categories. The first category is the multi-class approach, in which the entire feasible VOT range is divided into several predetermined intervals according to a discrete VOT distribution or some socio-economic characteristics. In an elastic demand multi-class network equilibrium model proposed by Yang et al. (2002), the feasible range of VOT is divided into a predetermined number of intervals of equal length based on different income levels; the entire population of trips is segmented accordingly into different groups with corresponding group-specific demand functions. The Frank–Wolfe algorithm was extended to solve the problem and the numerical results based on a simple test network highlighted the importance of incorporating user heterogeneity in toll road modeling. Other examples of this approach can be found in Mekky (1995, 1997) and Nagurney and Dong (2002).

The second category considers VOT to be continuously distributed across the population of trips. Leurent (1993) was among the first to propose a cost versus time (CVT) network equilibrium model for road pricing applications; such equilibrium is achieved when every trip-maker is assigned a path that minimizes his/her own generalized cost. The method of successive averages (MSA) was adapted to solve for the CVT equilibrium. Dial (1996) developed a static bi-criterion user equilibrium traffic assignment model with continuous VOT to forecast path choice and associated total arc flows in the presence of tolled alternatives. This model essentially can be reduced to a variational inequality (VI) problem and solved by the solution algorithm proposed in Dial (1997). Note that Leurent's CVT equilibrium model considered elastic demand and allowed only one criterion (travel time) to be flow dependent; while Dial's model assumed fixed demand and allowed both criteria to be flow-dependent. Additionally, the CVT model is a finite dimensional approach that takes path flows as variables unlike Dial's infinite dimensional model that uses link flows as variables.

Marcotte and Zhu (1997) considered the problem of determining an equilibrium state resulting from the interaction of infinitely many classes of customers, differentiated by a continuously distributed class-specific parameter. Solutions to an infinite dimensional VI problem were used to describe the equilibrium and obtained by a linearization algorithm, an infinite dimensional extension of the Frank–Wolfe algorithm. Marcotte (1999) suggested a unified algorithmic framework that iteratively solves the parametric shortest path problem (PSPP) and performs a line search in the descent direction, pointing out that solving the PSPP approximately by selecting parameter values in a suitable manner could allow solving the bi-criterion problem as efficiently as the single criterion problem. For a thorough review and comparison of previous studies on multi-class and multi-criterion network equilibrium models readers may refer to Nagurney and Dong (2002).

This study focuses on the more general situation of heterogeneous trip-makers with a continuous VOT distribution, to more realistically capture travelers' path choice behavior in response to toll charges. Although this critical issue of user heterogeneity has been considered in the aforementioned literature, all those network equilibrium assignment models were developed only for flat (or static) road pricing schemes, rather than dynamic (or time-dependent) ones. In fact, the attempt to accurately design and evaluate dynamic pricing schemes relies on a realistic representation of complex traffic dynamics and spatial and temporal vehicular interactions in network equilibrium assignment models, hence necessitating the extension of the heterogeneous traffic assignment model from the static regime to the DTA context. To this end, this paper presents a DTA model and its solution algorithm for the bi-criterion dynamic user equilibrium (BDUE) problem that allows heterogeneous users with different VOT preferences. The BDUE problem is formulated as an infinite dimensional VI, and solved by a simulation-based heuristic approach that embeds (i) a bi-criterion time-dependent least generalized cost path algorithm to obtain the set of extreme efficient paths and the corresponding breakpoints that partition the entire VOT interval and hence naturally define the multiple user classes, and (ii) a traffic simulator to describe the traffic flow propagation and determine experienced path travel costs for any given path flow pattern. Moreover, to circumvent the difficulty of storing the memory-intensive path set and routing policies for large-scale network applications, a vehicle-based implementation technique is proposed to use the vehicle path set as a proxy for keeping track of the path assignment results.

This paper is structured as follows. The next section gives the assumptions, definition and problem statement of the bi-criterion dynamic user equilibrium (BDUE) problem, followed by the mathematical model of the BDUE problem in Section 3. A simulation-based BDUE algorithmic framework is proposed in Section 4, together with a vehicle-based implementation technique. Section 5 presents the experimental results illustrating the convergence behavior of the algorithm and how the user heterogeneity affects the path flow pattern and toll road usage under different dynamic road pricing scenarios. Concluding remarks are in Section 6.

2. Assumptions, definitions and problem statement

Variables and notation

o subscript for an origin node, $o \in O \subseteq N$
d subscript for a destination node, $d \in D \subseteq N$
τ superscript for a departure time interval, $\tau = 1, \ldots, T$
α value of time (VOT), $\alpha \in [\alpha^{\min}, \alpha^{\max}]$
$P(o, d, \tau)$ the set of all feasible paths for a given triplet (o, d, τ)
p subscript for a path $p \in P(o, d, \tau)$
$r^{\tau}_{od}(\alpha)$ number of trips with VOT α departing from o to d in time interval τ
r^{τ}_{odp} number of trips departing from node o to node d in time interval τ, and $r^{\tau}_{od} = \int_{\alpha^{\min}}^{\alpha^{\max}} r^{\tau}_{od}(\alpha)\mathrm{d}\alpha$
$r^{\tau}_{odp}(\alpha)$ number of trips with VOT α departing from o to d in time interval τ that are assigned to path $p \in P(o, d, \tau)$
$r(\alpha)$ the class-specific time-dependent OD path flow vector for the trips with VOT α; $r(\alpha) = \{r^{\tau}_{od}(\alpha), \forall o, d, \tau, \text{and } p \in P(o, d, \tau)\}$
$\Omega(\alpha)$ the feasible set of $r(\alpha)$; i.e. $\Omega(\alpha) = \{r(\alpha)\}$
r the multi-class time-dependent OD path flow vector for the trips with all possible values of time; i.e. $r = \{r(\alpha), \forall \alpha \in [\alpha^{\min}, \alpha^{\max}]\}$
Ω the feasible set of r; $\Omega = \{r\} = \{\Omega(\alpha), \forall \alpha \in [\alpha^{\min}, \alpha^{\max}]\}$
$TT^{\tau}_{odp}(r)$ experienced path travel time for all trips departing from o to d at time τ assigned to path $p \in P(o, d, \tau)$
TC^{τ}_{odp} experienced path travel cost for all trips departing from o to d at time τ assigned to path $p \in P\, P(o, d, \tau)$
$G^{\tau}_{odp}(\alpha, r)$ experienced path generalized travel cost for trips with VOT α departing from o to d at time τ assigned to path $p \in P(o, d, \tau)$
f^{τ}_{odp} number of trips departing from o to d in time interval τ assigned to path $p \in P(o, d, \tau)$, and $f^{\tau}_{odp} = \int_{\alpha^{\min}}^{\alpha^{\max}} r^{\tau}_{odp}(\alpha)\mathrm{d}\alpha$
f the total time-varying path flow vector; i.e. $f \equiv \{f^{\tau}_{odp}, \forall o, d, \tau, \text{and } p \in P(o, d, \tau)\}$
F the feasible set of f. $F = \{f\} = \int_{\alpha^{\min}}^{\alpha^{\max}} \Omega(\alpha)\mathrm{d}\alpha$

Given a time-dependent network $G = (N, A)$, where N is the set of nodes and A is the set of directed links (i, j), $i \in N$ and $j \in N$. The time period of interest (planning horizon) is discretized into a set of small time intervals, $S = \{t_0, t_0 + \sigma, t_0 + 2\sigma, \ldots, t_0 + M\sigma\}$, where t_0 is the earliest possible departure time from any origin node, σ is a small time interval during which no perceptible changes in traffic conditions and/or travel cost occur, and M is a large number such that the intervals from t_0 to $t_0 + M\sigma$ cover the planning horizon S. Without loss of generality, associated with each arc (i, j) and time interval t are two essential time-dependent arc travel impedances: time ($d_{ij}(t)$) and cost ($c_{ij}(t)$), which are required to travel from node i to node j when departing at time t from node i, and would be minimized in trip-makers' path choice decision framework. Note that $d_{ij}(t)$ may include both non-congested travel time and delay, while some other cost-related arc attributes can be considered in $c_{ij}(t)$.

By assuming path travel disutilities are additive of their respective link travel disutilities, $TT^{\tau}_{odp} = \sum_{(i,j,t) \in p} d_{ij}(t)$ and $TC^{\tau}_{odp} = \sum_{(i,j,t) \in p} c_{ij}(t)$. The experienced generalized cost perceived by the trip-makers with VOT α departing from o to d at departure time τ along path $p \in P(o, d, \tau)$ is defined as:

$$G^{\tau}_{odp}(\alpha) = \sum_{(i,j,t) \in p} [c_{ij}(t) + \alpha \times d_{ij}(t)] = TC^{\tau}_{odp} + \alpha \times TT^{\tau}_{odp} \tag{1}$$

The VOT α relative to each trip represents how much money the trip-maker is willing to trade for a unit time saving. To reflect heterogeneity of the population, the VOT in this study is treated as a continuous random variable distributed across the population of trip-makers, with the density function $\phi(\alpha) > 0$, $\forall \alpha \in [\alpha^{\min}, \alpha^{\max}]$ and $\int_{\alpha^{\min}}^{\alpha^{\max}} \phi(\alpha)\mathrm{d}\alpha = 1$, where the feasible range of VOT is given by the closed interval $[\alpha^{\min}, \alpha^{\max}]$. Note that the distribution of VOT is assumed known, and can be estimated from survey data (e.g. Ben-Akiva et al., 1993; Hensher, 2001; Small et al., 2005). The time-dependent origin-destination (OD) demand for the entire feasible range of VOT over the planning horizon (i.e. $r^{\tau}_{od}(\alpha), \forall o, d, \tau$, and $\alpha \in [\alpha^{\min}, \alpha^{\max}]$) is also known a priori (in practice, the OD pattern and the VOT distribution will be considered independent of each other).

The key behavioral assumption made for the path choice decision is as follows: in a disutility-minimization framework, each trip-maker chooses a path that minimizes the generalized cost (i.e. disutility) function (1). Specifically, for trip-makers with VOT α, a path $p* \in P(o, d, \tau)$ will be selected if and only if $G^{\tau}_{odp*}(\alpha) = \min_{p \in P(o,d,\tau)} G^{\tau}_{odp}(\alpha)$. This assumption implies that a deterministic path choice approach is applied, though extension to include a stochastic path choice model (e.g. logit, probit or mixed variants) intended to represent travelers' travel time perception errors is also possible.

Based on this assumption, the bi-criterion dynamic user equilibrium (BDUE), a bi-criterion and dynamic extension of Wardrop's first principle, is defined as:

> For each OD pair and for each departure time interval, every trip-maker cannot decrease the experienced generalized trip cost with respect to that trip's particular VOT α by unilaterally changing paths.

This implies that, at BDUE, each trip-maker is assigned to a path having the time-dependent least generalized cost with respect to his/her own VOT. This definition can be also viewed as the dynamic extension of Dial's bi-criterion equilibrium traffic assignment (Dial, 1996) or Leurent's cost versus time equilibrium (Leurent, 1993).

Given the assumptions and definition above, this study aims at solving the BDUE traffic assignment problem, under a given time-dependent road pricing scheme, to obtain the time-dependent path flow pattern satisfying the BDUE condition. Specifically, the focus is on determining the BDUE path flows (routing policies) in a vehicular network for each OD pair, each departure time interval and all possible values of time, i.e. $r \equiv \{r^{\tau}_{odp}(\alpha), \forall o, d, \tau, p \in P(o, d, \tau)$ and $\forall \alpha \in [\alpha^{\min}, \alpha^{\max}]\}$. Note that the toll road usage can also be found by summing the path flows over all the paths using toll links.

3. Bi-criterion dynamic user equilibrium model

Since trips with different VOT (now a continuously distributed random variable) are assigned onto the same road network, the generalization of the classical dynamic user equilibrium traffic assignment allows a large number of classes of trips to be in a simultaneous equilibrium. In the extreme case where each possible value of time corresponds to a class of trips, infinitely many classes of trips would need to be assigned in a network. In other words, solving for the BDUE is equivalent to determining an equilibrium state resulting from the interactions of (possibly infinitely) many classes of trips in a network. Their interactions can be mainly reflected by assuming each of the (measured or actual) time-dependent path travel time functions $TT^{\tau}_{odp}(r), \forall o, d, \tau$, and $p \in P(o, d, \tau)$, as a function of $r \equiv \{r(\alpha), \forall \alpha \in [\alpha^{\min}, \alpha^{\max}]\}$, whereas time-dependent path travel costs (e.g. tolls) TC^{τ}_{odp} are assumed flow independent and known (though extensions of the formulation to consider flow-dependent prices are possible). By definition, the generalized path travel cost perceived by trips with VOT α: $G^{\tau}_{odp}(\alpha, r) = TC^{\tau}_{odp} + \alpha \times TT^{\tau}_{odp}(r)$ also depends on r. Although the multi-class path flow vector r might have infinitely many components, the corresponding total time-dependent OD path flow vector f is a finite dimensional subset of R^m, where $m = \sum_o \sum_d \sum_{\tau} |P(o, d, \tau)|$. In this paper, time-dependent generalized path costs $G^{\tau}_{odp}(\alpha, r)$ and $G^{\tau}_{odp}(\alpha, f)$ are used interchangeably.

Let $\Omega(\alpha) \equiv \{r(\alpha)\}$ be the feasible set of path flow vectors $r(\alpha)$ satisfying the path flow conservation and non-negativity constraints:

$$\sum_{p \in P(o,d,\tau)} r^{\tau}_{odp}(\alpha) = r^{\tau}_{od}(\alpha), \quad \forall o, d, \text{ and } \tau, \tag{2}$$

$$r^{\tau}_{odp}(\alpha) \geqslant 0, \quad \forall o, d, \tau, \text{ and } p \in P(o, d, \tau). \tag{3}$$

The following Lemma gives the VI formulation of the BDUE problem of interest.

Lemma 1. *Solving for the BDUE flow pattern r^* is equivalent to finding the solution of a system of variational inequalities $r^*(\alpha) \in \Omega(\alpha)$ such that*

$$\sum_o \sum_d \sum_{\tau} \sum_{p \in P(o,d,\tau)} (TC^{\tau}_{odp} + \alpha \times TT^{\tau}_{odp}(r)) \times (r^{\tau}_{odp}(\alpha)^* - r^{\tau}_{odp}(\alpha)) \leqslant 0, \quad \forall r(\alpha) \in \Omega(\alpha), \alpha \in [\alpha^{\min}, \alpha^{\max}], \tag{4}$$

or in the following vector form for simplicity and clarity:

$$G(\alpha, r^*)^{\mathrm{T}} \circ (r^*(\alpha) - r(\alpha)) \leqslant 0, \quad \forall r(\alpha) \in \Omega(\alpha), \quad \forall \alpha \in [\alpha^{\min}, \alpha^{\max}], \tag{5}$$

where $\circ$ *is the inner product in* R^m, *and* $G(\alpha, r^*)$ *is the path generalized cost vector perceived by the trips with VOT* α *and evaluated at flow pattern* r^*. *Since* (4) or (5) *is only required to hold on* $[\alpha^{\min}, \alpha^{\max}]$, *it can be further represented by the following (possibly) infinite dimensional VI (e.g.* Marcotte and Zhu, 1997*): find* $r^* \equiv \{r^*(\alpha), \forall \alpha \in [\alpha^{\min}, \alpha^{\max}]\}$ *and* $r^* \in \Omega$ *such that*

$$G(r^*)^{\mathrm{T}} \circ (r^* - r) \leqslant 0, \quad \forall r \in \Omega \tag{6}$$

where $G(r^*) \equiv \{G(\alpha, r^*), \forall \alpha \in [\alpha^{\min}, \alpha^{\max}]\}$. *Note that* $G(r^*)$ *and* r^* *(or* r*) have the same (possibly infinite) number of elements. The following proof of* Lemma 1 *adapts the results from* Smith (1979).

Proof of Lemma 1. Suppose r^* satisfies the BDUE condition, and let $G(r^*)$ be the corresponding path generalized cost vector. According to the BDUE definition, for an equilibrium multi-class path flow vector r^*, the following condition can be established:

$$\begin{aligned} &\text{If } G^\tau_{odq}(\alpha, r^*) > G^\tau_{odp}(\alpha, r^*)(= \pi^\tau_{od}(\alpha)), \quad \text{then } r^\tau_{odq}(\alpha) = 0. \\ &\text{If } G^\tau_{odq}(\alpha, r^*) = G^\tau_{odp}(\alpha, r^*)(= \pi^\tau_{od}(\alpha)), \quad \text{then } r^\tau_{odq}(\alpha) > 0. \\ &\quad \forall o, d, \tau, p \quad \text{and } q \in P(o, d, \tau), \quad \text{and} \quad \forall \alpha \in [\alpha^{\min}, \alpha^{\max}] \end{aligned} \tag{7}$$

where $\pi^\tau_{od}(\alpha)$ is the minimum possible generalized travel cost for the trips with VOT α from o to d departing at time τ.

Consider these path generalized costs $G(r^*)$ as fixed at the current level of path flow r^*. Because r^* satisfies the BDUE condition and hence only least generalized cost paths are used, total generalized cost cannot be reduced by moving flows from least generalized cost paths to other inefficient paths. For instance, if path generalized costs are fixed as $G(r^*)$ and $G^\tau_{odq}(\alpha, r^*) > G^\tau_{odp}(\alpha, r^*)$ then moving flow $r^\tau_{odp}(\alpha)$ from p to q will lead to an increase of total generalized cost by $r^\tau_{odp}(\alpha) \times G^\tau_{odq}(\alpha, r^*) - r^\tau_{odp}(\alpha) \times G^\tau_{odp}(\alpha, r^*) > 0$. Therefore any other feasible multi-class path flow vector $r \in \Omega$ has total generalized cost at least as large as r^* which uses only cheapest paths. In other words, the BDUE path flow vector r^* satisfying (7) is also the solution of the infinite dimensional variational inequality (6):

$$G(r^*)^{\mathrm{T}} \circ r^* \leqslant G(r^*)^{\mathrm{T}} \circ r \text{ or } G(r^*)^{\mathrm{T}} \circ (r^* - r) \leqslant 0, \quad \forall r \in \Omega \tag{8}$$

Conversely, assume that condition (7) does not hold, then there exists the following situation for some triplet (o, d, τ) and paths p and q : $r^\tau_{odq}(\alpha) > 0$ and $G^\tau_{odq}(\alpha, r^*) > G^\tau_{odp}(\alpha, r^*)(= \pi^\tau_{od}(\alpha))$. Moving flow $r^\tau_{odq}(\alpha)$ from q to the cheaper path p will result in a reduction of total generalized cost by $r^\tau_{odq}(\alpha) \times G^\tau_{odq}(\alpha, r^*) - r^\tau_{odq}(\alpha) \times G^\tau_{odp}(\alpha, r^*) > 0$. Let the resulting flow pattern be $r \in \Omega$. Then $G(r^*)^{\mathrm{T}} \circ r < G(r^*)^{\mathrm{T}} \circ r^*$, in which case (6) is not satisfied.

From the above, if (7) is satisfied then (6) is satisfied, and if (7) does not hold then (6) does not, either. Thus, conditions (7) and (6) are equivalent, and solving for the BDUE flow pattern is equivalent to finding the solution of the (possibly) infinite dimensional variational inequality (6). This completes the proof.

Although the theoretical guarantee of properties such as existence, uniqueness, and stability of the solution to infinite dimensional variational inequality (6) can be analytically derived, it generally requires the generalized path cost function, i.e. $G(\alpha, r)$ (or $G(\alpha, f)$), to be continuous and strictly monotone (see e.g. Marcotte and Zhu, 1997). These properties of path cost functions may not be satisfied in general road networks with complex traffic controls. Moreover, when path travel times are determined by traffic simulation, with adaptive signal control and realistic junction modeling, these properties cannot be established analytically. It may be useful to construct systematic simulation experiments to investigate whether conditions could be identified under which such properties can be expected to hold—however this is beyond the scope of the present study. □

4. The simulation-based BDUE solution algorithm

The BDUE solution algorithm proposed in this study is a bi-criterion extension of the simulation-based DTA heuristic approach developed by Peeta and Mahmassani (1995). Although the mathematical abstraction of the problem is a typical analytical formulation (i.e. a system of variational inequalities), to tackle many practical aspects of the DTA planning and operational applications (Peeta and Ziliaskopoulos, 2001), this study adopts the simulation-based solution approach, in which the critical constraints that describe the traffic flow propagation and the spatial and temporal interactions (e.g. flow conservation and vehicle movements) are addressed through the traffic simulation instead of analytical calculations. The flow chart of this approach is presented in Fig. 1.

4.1. Steps of the simulation-based BDUE algorithm

Note that in the following description of its functional steps the iteration subscript k of variables is dropped for simplicity and clarity of presentation.

4.1.1. Step 0: Input and initialization

Set iteration counter $k = 0$. Four key inputs for this algorithm include: (1) a time-dependent origin-destination (OD) demand matrix for the entire feasible range of VOT over the planning horizon ($r^{\tau}_{od}(\alpha), \forall o, d, \tau$, and $\alpha \in [\alpha^{\min}, \alpha^{\max}]$), (2) a time-dependent link toll vector, (3) VOT density and distribution functions ($\phi(\alpha)$ and $\Phi(\alpha)$, $\forall \alpha \in [\alpha^{\min}, \alpha^{\max}]$), and (4) initial paths and path flows.

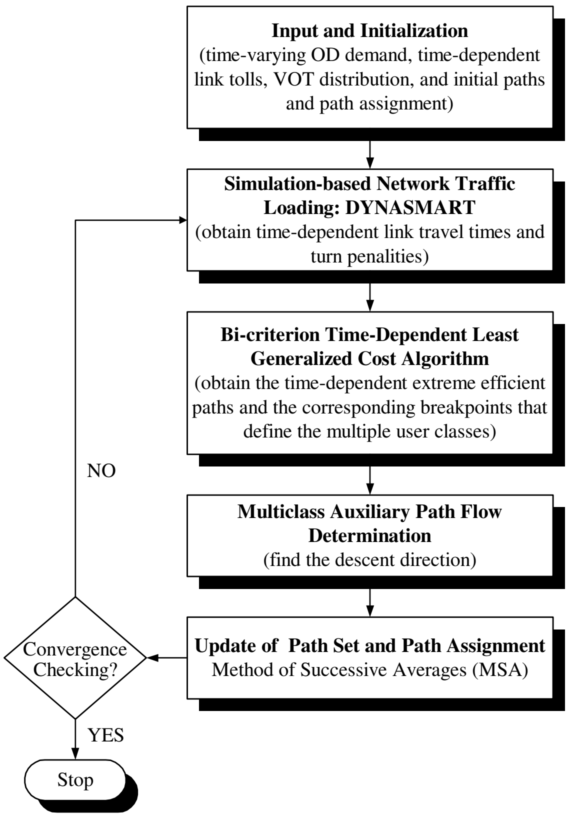

Fig. 1. Flow chart of the simulation-based BDUE algorithm.

4.1.2. Step 1: Simulation-based network traffic loading

Given the set of extreme efficient paths ($P^{ex}(o, d, \tau)$, $\forall o, d, \tau$) and the corresponding path assignments or routing policies ($r \equiv \{r(\alpha), \forall \alpha \in [\alpha^{min}, \alpha^{max}]\}$), the traffic simulation model DYNASMART (Jayakrishnan et al., 1994; Mahmassani, 2001) is used to evaluate the path assignments r, and to obtain the resulting time-dependent link travel times. Those travel times, together with the given set of time-dependent link tolls, are the input of the subsequent direction finding step. DYNASMART uses a hybrid (mesoscopic) approach to capture the dynamics of vehicular traffic flow in the simulation, whereby vehicles are moved individually according to prevailing local speeds, consistent with macroscopic flow relations on links. It should be noted that the algorithm is independent of the specific dynamic traffic model selected; any particle-based (microscopic or mesoscopic) dynamic traffic model capable of capturing complex traffic flow dynamics can be embedded into the proposed algorithm.

4.1.3. Step 2: Bi-criterion time-dependent least generalized cost path (BTDLGCP) algorithm

The main impediment for solving the BDUE problem of interest is due largely to the relaxation from a constant VOT α to a continuous random variable and hence the need to find an equilibrium state resulting from the interactions of (possibly infinitely) many classes of trips, each of which depends on the different VOT, in a network. If each class requires its own set of least generalized cost paths for all OD pairs and all departure time intervals, solving and storing such a grand path set is computationally intractable in applications on road networks with realistic sizes. Fortunately, in the disutility-minimization static bi-criterion traffic assignment model, all trips are distributed only among the (finite) set of extreme efficient (or non-dominated) paths corresponding to the extreme points on the efficient frontier in the criterion space (e.g. Dial, 1996; Marcotte and Zhu, 1997). Analogously, only a set of time-dependent extreme efficient paths, each of which minimizes the parametric function (1) for a particular VOT interval, needs to be determined in the bi-criterion dynamic traffic assignment model of interest.

This study applies the BTDLGCP algorithm developed by Mahmassani et al. (2005b) to find the complete set of time-dependent extreme efficient paths ($P^{ex}(o, d, \tau)$ $\forall o, d, \tau$), to which all the trips with different values of time are assigned, and the corresponding set of breakpoints (i.e. values of time: $\boldsymbol{\alpha} = \{\alpha^0, \alpha^1, \ldots, \alpha^I | \alpha^{min} = \alpha^0 < \alpha^1 < \cdots < \alpha^i < \cdots < \alpha^I = \alpha^{max}\}$) that partitions the entire feasible range of VOT $[\alpha^{min}, \alpha^{max}]$ and hence defines the multiple classes of trips, where each class u^i covers the trips with VOT $\alpha \in [\alpha^{i-1}, \alpha^i)$, $i = 1, \ldots, I$. Starting from the lowest possible VOT, the bi-criterion time-dependent shortest path algorithm continuously solves for the time-dependent least generalized cost (TDLGC) path tree rooted at each destination for a given VOT interval and determines the upper bound of that VOT interval, for which the TDLGC path tree remains optimal, until reaching the highest possible of VOT. It is important to note that the proposed BDUE algorithm is independent of the underlying VOT distribution specified; the only input, relevant to the VOT distribution, to the BTDLGCP algorithm is the feasible range of the VOT distribution. Although it is theoretically preferred to have a complete set of time-dependent extreme efficient paths, finding such a path set could be computationally prohibitive in large-scale network applications. Mahmassani et al. (2005b) proposed an approximation scheme, embedded in a binary search framework and utilizing the underlying VOT distribution, to generate only the partial (but representative in terms of expected approximation errors) set of time-dependent extreme efficient paths and the corresponding breakpoints.

4.1.4. Step 3: Direction finding: multi-class auxiliary path flow determination

Given the set of time-dependent extreme efficient paths solved by the BTDLGCP algorithm and the corresponding set of breakpoints defining the classes of trips, the (descent) search direction based on the multi-class auxiliary path flows is obtained as the follows. For each OD pair and each departure time interval, all the trips with the same (o, d, τ) and of class u^i are assigned to the time-dependent extreme efficient path $p(i) \in P^{ex}(o, d, \tau)$ corresponding to the VOT subinterval $[\alpha^{i-1}, \alpha^i)$. The number of trips using path $p(i)$ is determined by

$$y^{\tau}_{odp}(u^i) = r^{\tau}_{od} \times \left[\int_{\alpha^{i-1}}^{\alpha^i} \phi(\alpha) \mathrm{d}\alpha\right] = r^{\tau}_{od} \times [\Phi(\alpha^i) - \Phi(\alpha^{i-1})] \tag{9}$$

where $\Phi(\alpha)$ is the cumulative distribution function of random variable α. Eq. (9) is then applied to each class u^i, $\forall i = 1, \ldots, I$. The multi-class path flow vector $y \equiv \{y(u^i),\ i = 1, \ldots, I\}$ where $y(u^i) \equiv \{y^{\tau}_{odp}(u^i),$ $\forall o, d, \tau,$ and $p(i) \in P^{\mathrm{ex}}(o, d, \tau)\}$, is used as the auxiliary solution in the search process, and the descent direction is obtained as: $d = y - r$. Note that the auxiliary path flow vector y might be viewed as the result of a multi-class dynamic network assignment in the sense that all the trips with VOT $\alpha \in [\alpha^{i-1}, \alpha^i)$ are assigned to the corresponding path $p(i)$, $\forall i = 1, \ldots, I$.

Recall that in applying the convex combinations method (Sheffi, 1985; Patriksson, 1994), such as the Frank–Wolfe algorithm, to the static traffic assignment problem, the user equilibrium solution can be expressed by the convex combinations of a finite set of extreme points successively obtained by the all-or-nothing assignment (i.e. auxiliary solutions) in the previous iterations. Analogously, by assuming that the total path flow polyhedron F is a convex and compact set, the BDUE traffic assignment problem of interest can be solved by generating a sequence of multi-class auxiliary path flow vectors to approach the BDUE flow pattern. Specifically, each multi-class auxiliary path flow vector y contributes, through the path set and path assignments updating mechanism described in the next step, a portion to the BDUE flow pattern r^* of interest. Moreover, determining the multi-class auxiliary path flow vector in the step of descent direction finding (i.e. solving for the (finite) set of time-dependent extreme efficient paths and assigning the corresponding trips to each of them) circumvents the difficulty of dealing with the infinite dimensional variational inequality.

4.1.5. Step 4: Update of path set and path assignments – method of successive averages (MSA)

Rather than applying a one-dimensional line search algorithm to solve for the move size γ_k along the descent searching direction d_k of iteration k, this study adopts a predetermined move size in the solution algorithm, known as the method of successive averages (MSA). The sequence of move sizes has to satisfy the following conditions: $\sum_{k=1}^{\infty}\gamma_k = \infty$ and $\sum_{k=1}^{\infty}\gamma_k^2 < \infty$, for an algorithm to converge. One of the most convenient move size satisfying the conditions above is the reciprocal of iteration number; i.e. $\gamma_k = 1/k$. With this move size, the updating of path assignments r_k of iteration k to obtain the path assignments r_{k+1} for iteration $k + 1$ is performed as the following:

$$r_{k+1} = r_k + 1/k \times d_k = r_k + 1/k \times (y_k - r_k) = (1 - 1/k) \times r_k + 1/k \times y_k \tag{10}$$

where y_k is the auxiliary path assignments obtained in the direction finding step of iteration k (i.e. MDNL). Although the use of predetermined move size from MSA may lack search efficiency, it reduces the computational efforts for analytically/numerically optimizing the move size.

4.1.6. Step 5: Convergence checking

The convergence criterion proposed by Peeta (1994) is applied and this criterion is based on the comparison of path assignments over successive iterations. Specifically, for each $(o, d, \tau, p(i))$, the number of vehicles assigned to a path $p(i)$ in the current iteration k (i.e. $r^{\tau}_{odp(i),k}$) is compared with that in the next iteration $k + 1$ (i.e. $r^{\tau}_{odp(i),k+1}$). The number of cases, in which the absolute difference between $r^{\tau}_{odp(i),k}$ and $r^{\tau}_{odp(i),k+1}$ is greater than the predetermined threshold ε, is recorded as $N(\varepsilon)$. If $N(\varepsilon) < \varphi$, a predetermined upper bound, convergence is assumed. The program terminates and r_{k+1} is the BDUE solution.

4.2. Implementation considerations for large-scale network applications

The above BDUE algorithm is featured as a path-based approach, necessitating the explicit storage of the path set and path assignment results for each (o, d, τ). Although it is straightforward to record all the paths and the corresponding path choice probabilities for each (o, d, τ) by using multi-dimensional arrays, computer memory requirements grow dramatically when the number of OD pairs is large, or many iterations are required to achieve convergence. Furthermore, the relaxation to the continuously distributed VOT requires a set of extreme efficient paths $P(u^i) = \{P(u^i, o, d, \tau), \forall o, d, \tau\}$ for each VOT subinterval, and the number of subintervals is unknown a priori and changes from iteration to iteration, making it more difficult to construct a memory efficient data structure for storing and updating the huge path set and path assignments in large-scale network applications. Essentially, as an attempt to accommodate greater behavioral and policy realism in applying DTA models for designing and evaluating dynamic pricing schemes, modeling heterogeneous users with a range of VOT as opposed to identical users exacerbates the computational complexity and memory requirement.

In a particle-based and simulation-based DTA system, vehicles carry their paths from iteration to iteration, and the vehicle path set implicitly reflects and stores the path set and path assignments results. This is particularly advantageous for large-scale DTA applications, as the total number of feasible paths generated by the iterative solution algorithm, after a certain number of iterations, could be significantly greater than the total number of vehicles, which is determined a priori by the OD demand table. For example, in the Portland transportation planning network (Nagel et al., 2000), there are about 1,260 traffic analysis zones (TAZ) and 1.5 million OD pairs, and the corresponding total trip-makers are 1.5 million in all time periods. Obviously, every OD pair requires more than one time-dependent shortest path for reaching the dynamic network user equilibrium. Thus, storing the vehicle path set is more memory efficient than storing the complete path set and routing policies for large-scale networks, and one can circumvent the need for explicitly storing the path set and routing policies by applying the following efficient path assignment updating technique.

According to the rightmost part of Eq. (10), $(1 - 1/k)$ of current path assignments r_k are kept to the next iteration, while $1/k$ of the auxiliary path assignment y_k are added to the updated path assignments for next iteration r_{k+1}. That is, in a given iteration k, for each class u^i corresponding to VOT subinterval $[\alpha^{i-1}, \alpha^i)$, only $1/k$ of the corresponding vehicles, are moved to the auxiliary path $p(i)$ found in this iteration, and the remaining vehicle paths keep the same. Essentially, this implementation technique uses the vehicle path set as a proxy for the exact path set and assignment results, and the path set and routing policies of interest can be approximately recovered from the realized vehicle paths in the last iteration's simulation results.

5. Numerical experiments

Numerical experiments are conducted on several real road networks to examine the BDUE algorithm. In addition to the algorithmic convergence property, with the explicit consideration of user heterogeneity, of particular interest is how the VOT distribution affects the path flow patterns and toll road usage under different dynamic pricing scenarios. The proposed BDUE algorithm is implemented using the aforementioned vehicle-based technique, which can be seamlessly integrated with any mesoscopic/microscopic dynamic traffic simulation model and is considered particularly appealing for large network deployment of DTA models. The algorithm is coded and compiled by using the Compaq Visual FORTRAN 6.6 and evaluated on the Windows XP platform and a machine with an Intel Pentium IV 2.8 GHz CPU and 4 GB RAM.

In all experiments conducted, the following parameter settings are applied. The continuous VOT distribution considered in the experiments is a normal distribution with (mean, standard deviation) = (20, 10), which has the unit USD/Hour and is denoted as $N(20, 10)$. The parameters of this normal distribution are set according to the measurements from a value pricing experiment conducted by Lam and Small (2001), and also similar to those used by Yang et al. (2002). The feasible range of the VOT distribution $[\alpha^{\min}, \alpha^{\max}]$ is [0.5, 300]. The resolution (aggregation interval) of the bi-criterion time-dependent least generalized cost path tree calculation is set to 0.1 min, which is the same as the time step for the simulation. The OD demand assignment interval (or departure time interval) is set to 1 min. The convergence threshold φ is set to 100; the algorithm will terminate if there are less than 100 vehicles changing paths between two successive iterations. The initial solutions of the experiments are obtained by loading time-varying OD demands to their respective time-dependent least generalized cost paths calculated based on the prevailing travel times output from the traffic simulator.

Another measure of effectiveness collected in the experiments is Gap(r):

$$\mathrm{Gap}(r) = \sum_{u^i} \sum_{o} \sum_{d} \sum_{\tau} \sum_{p \in P(u^i,o,d,\tau)} r^{\tau}_{odp}(u^i) \times [G^{\tau}_{odp}(u^i, r) - \pi^{\tau}_{od}(u^i, r)] \tag{11}$$

where $r^{\tau}_{odp}(u^i)$ is the number of class u^i trips with VOT $\alpha \in [\alpha^{i-1}, \alpha^i)$ departing from o to d in time interval τ that are assigned to path $p \in P(u^i, o, d, \tau)$, $G^{\tau}_{odp}(u^i, r)$ is the generalized path cost of class u^i trips departing from o to d in time interval τ that are assigned to path $p \in P(u^i, o, d, \tau)$, and $\pi^{\tau}_{od}(u^i, r)$ is the least generalized cost of class u^i trips departing from o to d in time interval τ, evaluated at the path assignment r. Gap(r^l) provides a measure of the violation of the BDUE conditions in terms of the difference between the total actual experienced path generalized cost and the total least generalized cost evaluated at any given multi-class path flow pattern r. The difference vanishes when the path flow vector r^* satisfies the BDUE conditions. Essentially, the smaller the gap, the closer the solution is to a BDUE.

Note that this study aims at developing a bi-criterion DTA model for evaluating dynamic pricing scenarios but not solving for a toll vector that improves local or network-wide performance. Hence, testing different dynamic toll vectors in the conducted experiments does not intend to compare their effectiveness on reducing congestion, and focuses exclusively on demonstrating, in evaluating *given* dynamic road pricing scenarios, what the BDUE model can accomplish and why the user heterogeneity should be addressed.

5.1. Experiments on the irvine network

The Irvine (California, USA) network depicted in Fig. 2 consists of 326 nodes (70 of them are signalized), 626 links, and 61 traffic analysis zones (TAZ) and had been calibrated by using real-world observations from multiple-day detector data (Mahmassani et al., 2003). An available 2-hour (7–9 AM) morning peak time-varying OD demand table is extracted and loaded to the network, with 35,304 vehicles in the observation period (7:10–8:50 AM). To create hypothetic dynamic road pricing scenarios, one lane of a portion (about 1 mile) of the I-405 westbound freeway is converted to the toll road, along with an additional new toll lane. The two toll lanes have the same length as the (remaining) three regular lanes but a 10-mile higher posted speed limit (and hence higher capacity) than the regular lanes. Table 1 lists the three simple dynamic pricing scenarios tested in the experiments conducted on the Irvine network. These three pricing scenarios have the same four pricing periods but different toll levels representing low, middle, and high toll scenarios, respectively.

The experimental results are presented in Table 2. It can be found that the BDUE algorithm converges in all three pricing scenarios tested on the Irvine network, although the convergence patterns in term of total violations are not strictly monotonic decreasing. To examine the solution quality of the BDUE algorithm, the iteration-by-iteration gap values for all three scenarios are also reported in the same table. The final gap values obtained by the BDUE algorithm are 15.7% (262.7/1669.0), 16.2% (299.4/186.6), and 13.0%

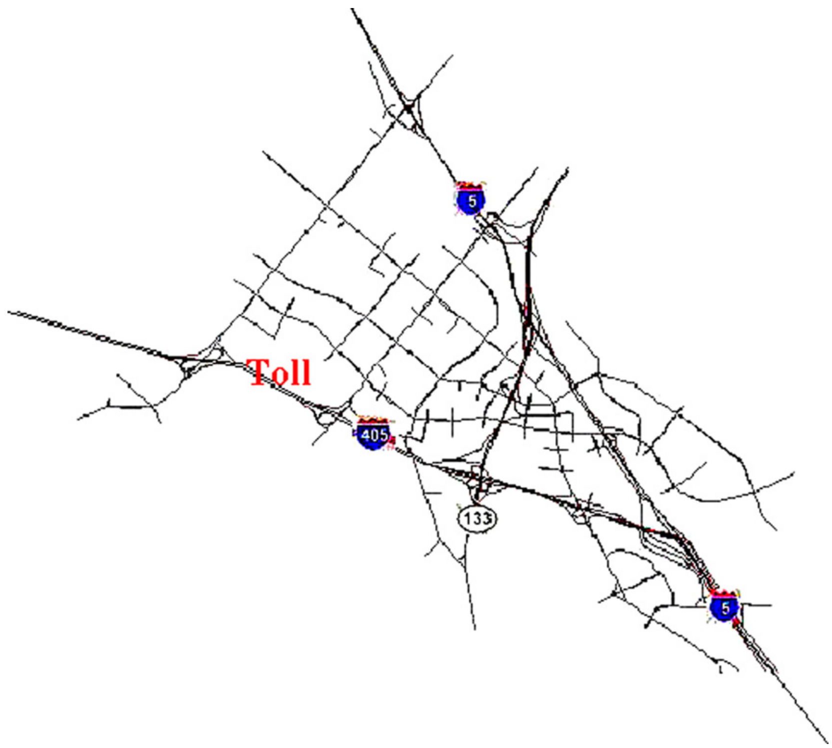

Fig. 2. The Irvine network with hypothetic toll road.

Table 1
Dynamic road pricing scenarios tested on the Irvine network

Pricing scenario	Period 1 (7:00–7:30 AM)	Period 2 (7:30–8:00 AM)	Period 3 (8:00–8:30 AM)	Period 4 (8:30–9:00 AM)
1	\$0.10	\$0.20	\$0.30	\$0.15
2	\$0.20	\$0.30	\$0.40	\$0.25
3	\$0.30	\$0.40	\$0.50	\$0.35

Table 2
Experimental results on the Irvine network

Iteration	Total violations			Gap(r)		
	Scenario 1	Scenario 2	Scenario 3	Scenario 1	Scenario 2	Scenario 3
1	15907.9	13372.4	12610.8	1669.0	1846.6	2224.7
2	8501.1	7172.7	6985.3	1478.6	1851.8	1444.9
3	5711.8	4931.0	4770.0	1260.1	737.3	873.0
4	3463.4	2799.2	2752.3	559.1	551.4	764.1
5	2273.7	1807.1	1743.4	536.4	1034.5	483.3
6	1515.6	1180.1	1036.3	485.9	657.2	614.2
7	1065.0	733.2	791.8	917.0	703.8	395.9
8	780.1	507.3	546.3	452.9	458.8	695.4
9	865.0	331.1	695.8	724.5	809.5	451.4
10	526.3	554.7	313.2	310.9	287.5	333.3
11	353.6	270.7	274.8	698.1	785.5	452.7
12	233.1	474.7	187.3	320.6	863.7	295.5
13	377.7	202.2	405.3	312.5	316.2	314.2
14	210.5	150.0	161.2	291.4	300.2	290.0
15	150.8	119.5	94.2	602.1	285.3	
16	231.9	254.0		406.1	263.2	
17	101.7	75.5		361.7	299.4	
18	70.1			262.7		

(290.9/2224.7) of the initial gap values, respectively, for the three different scenarios listed in Table 1. These small gap values indicate that the BDUE algorithm is able to find close-to-BDUE solutions (see Fig. 3).

To highlight the advantage of memory efficiency of the vehicle-based implementation technique, a grand path set version of the BDUE algorithm is also implemented by using fixed size arrays to store the complete extreme efficient path set and routing policies for all iterations. With identical experimental settings, the grand path set version is found to require more than 2.83 GB memory (the largest memory size available for a single 32-bit Windows application is 3.0 GB), while the vehicle path set version needs about 2.14 GB memory. Note that although some advanced data structures might be applied to reduce the memory usage of the grand path set version, this difference in memory usage is still proportional to the problem (or network) size and the number of iterations required to reach the convergence.

5.2. Experiments on the Baltimore–Washington, DC corridor network

To demonstrate the capability of the BDUE algorithm for large-scale networks with dynamic road pricing scenarios, this set of experiments is conducted on a recently coded large road network, the Baltimore–Washington, DC corridor (BW) network, which consists primarily of the I-95 freeway corridor between Washington, DC and Baltimore (Maryland, USA) and is bounded by two beltways (I-695 Baltimore Beltway on the north and I-495 Capital Beltway on the south). The BW network has 2241 nodes (231 of them are signalized), 3459 links and 111 traffic analysis zones (TAZ), and been calibrated by using real-world observations from multiple-day detector data (Mahmassani et al., 2005a). An available 1-hour (7:30–8:30 AM) morning peak time-varying OD demand (58,767 vehicles) table is extracted and loaded to the network. To create hypothetic dynamic toll scenarios, one of the 20-mile long southbound lanes of the I-95 corridor is converted to the toll road, together with an additional new toll lane. The two toll lanes have the same length, posted speed limit,

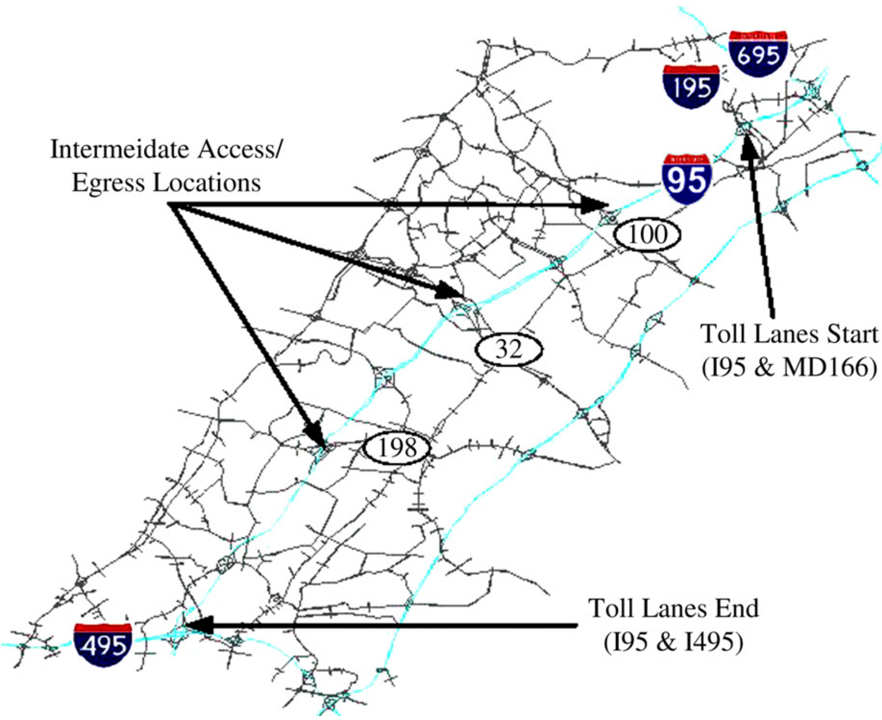

Fig. 3. The Baltimore–Washington, DC corridor network with hypothetic toll road.

and capacity as the (remaining) three regular lanes. The major merge points to the I-95 corridor are from I-195, MD-100, MD-32 and MD-198, where the toll road access/egress points are specified. A dynamic link toll vector generated by the method proposed by Dong et al. (2007) is used in this network to test the BDUE algorithm. The method solves for a vector of time-varying link tolls so as to maintain high level of service on the toll road. Essentially, the deviations between (prevailing or predicted) link concentrations and a given set of target concentrations on toll links are calculated, and then link tolls are determined by some control regulator according to the deviations.

The convergence pattern in terms of total violations of the algorithm is plotted in Fig. 4. As shown in this figure, the algorithm reaches the convergence criterion at the 22nd iteration. The initial and final gap values

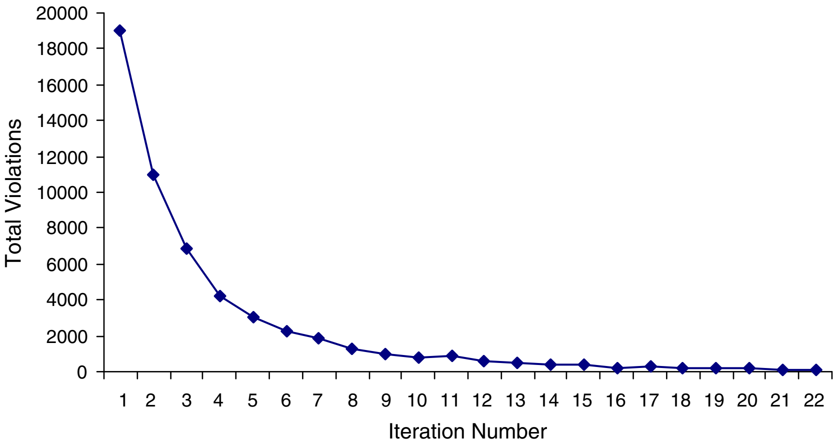

Fig. 4. The convergence pattern of the algorithm on the BW network.

are 3911.5 and 1254.7, respectively. The gap reduction is $(3911.5 - 1254.7)/3911.5 = 68\%$. The memory usage is 2.95 GB and the computation time of finishing 22 iterations is about 30 hours. The grand path set version of the algorithm fails in this experiment as it needs (much) more than 3.0 GB memory. This experiment further demonstrates the contribution of introducing the vehicle-based implementation technique in developing large-scale DTA network models for evaluating dynamic road pricing scenarios.

5.3. Experiments on the Fort Worth network

This set of experiments intends to investigate the impacts of user heterogeneity in terms of VOT on predicting path flow patterns and toll road usage in evaluating different dynamic road pricing scenarios. The Fort Worth (Texas, USA) network, depicted in Fig. 5 and consisting of 180 nodes (62 of them are signalized), 445 links and 13 traffic analysis zones (TAZ), is used in the experiments. An available 1-hour time-varying OD demand (23,000 vehicles) table is loaded to the network. The planning horizon is 90 minutes while the statistics are collected only from 10 to 50 minutes to take into account the time for simulation warm-up and network clearance. Other experimental parameters are kept the same as they are in the preceding two set of experiments. Similarly, one lane of a portion (about 1.5 miles) of the I-35 northbound freeway corridor is converted to the toll road, along with an additional new toll lane. The two toll lanes have the same length, posted speed limit, and capacity as the (remaining) three regular lanes. Table 3 lists the four simple dynamic pricing scenarios tested in this set of experiments.

In addition to the continuous $N(20, 10)$ VOT distribution, two other VOT cases are considered as well: one is a constant VOT equal to \$20/h, and the other one is a discrete VOT distribution in which the entire population is segmented into three groups according to different trip purposes (with mean VOT = \$20/h).

Group 1: commute trips, 50%, VOT = \$20/h;
Group 2: business trips, 25%, VOT = \$30/h;
Group 3: other trips, 25%, VOT = \$10/h.

Total violations and average trip time are reported as the figures of merit for examining the convergence pattern and overall network performance. Toll road usage is defined as the percentages of vehicles passing

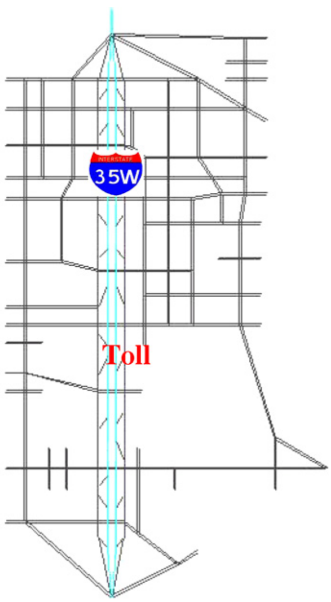

Fig. 5. The Fort Worth network with hypothetic toll road.

Table 3
Dynamic road pricing scenarios tested on the Fort Worth network

Pricing scenario	Period 1 (0–30 min)	Period 2 (30–60 min)
1	\$0.10	\$0.15
2	\$0.30	\$0.50
3	\$0.75	\$1.00
4	\$1.00	\$1.50

through toll links, and this measure is used to explore the impact of VOT distributions on network flow patterns under different given dynamic pricing scenarios.

The convergence pattern of the proposed BDUE algorithm with different VOT distributions under pricing scenario 1 is presented in Fig. 6. As shown in the figure, just like the convergence pattern of the classical Frank–Wolfe algorithm, the number of total violations decreases rapidly in the first few iterations and then slowly afterwards. Moreover, the result shows that the convergence patterns of the algorithm with different VOT distributions are similar, suggesting that the VOT distribution would not affect the performance (or convergence) of the algorithm. This result conforms to expectation because in each iteration, the algorithm first determines the multi-class auxiliary path flows using the bi-criterion time-dependent least cost path algorithm, and then updates path flow assignments for each user class using the MSA. The VOT distribution would only affect the determination of the multiple classes, and has a limited effect on the convergence pattern (in terms of the total violations) of the solution algorithm.

Fig. 7 provides the toll road usage over the planning horizon of one major OD pair using the northbound of the freeway corridor predicted by the BDUE model with different VOT distributions and under different pricing scenarios. As illustrated in this figure, when the toll charge is low (scenario 1), the toll road usage predicted by the DTA model with a single constant VOT is higher than that forecasted by the BDUE model with continuous or discrete VOT distributions. Since the single VOT model assumes homogeneous users, all users with this common VOT are willing to use the toll road when the toll charge is low. However, there are in fact a certain number of trips that have lower VOT and may not want to use the toll road even when the toll charge is not high. This phenomenon can be captured in the proposed BDUE model with continuous or discrete VOT by recognizing the existence of those low VOT users in the heterogeneous population. On the other hand, when the toll charge is high (scenario 4), the constant VOT model gives lower toll road usage than the continuous or discrete VOT model, because, in this case, it assumes that all users behave identically in response to the higher toll charge so travelers are less likely to use the toll road to save time. The BDUE model acknowledges the fact that there is a certain portion of high VOT trips that still wish to take the expensive but fast toll road (freeway). If the more realistic VOT distribution $N(20,10)$ is considered as the benchmark, then both

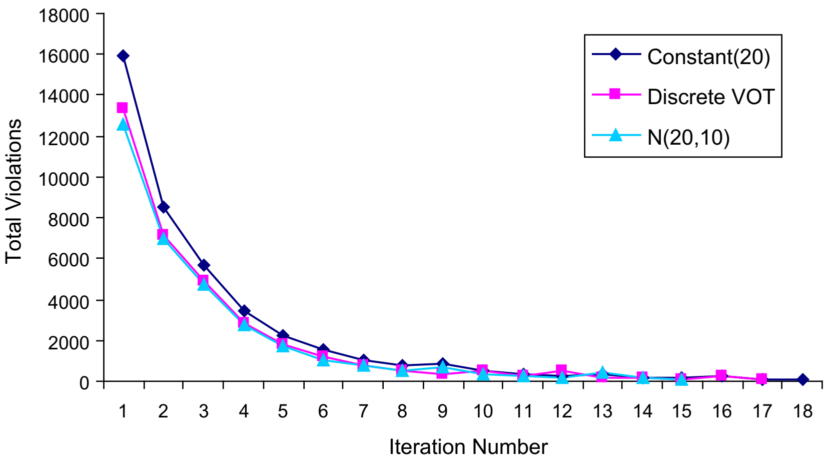

Fig. 6. Convergence patterns of the BDUE algorithm with different VOT distributions.

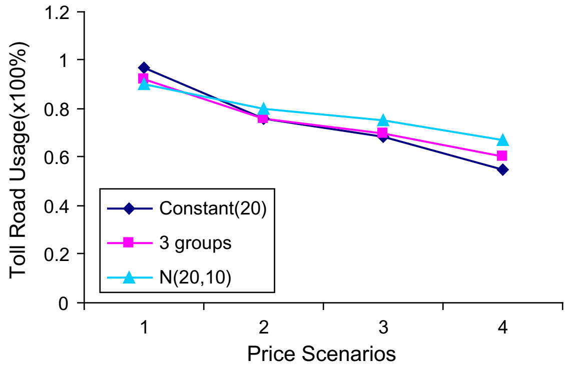

Fig. 7. Overall toll road usage under different pricing scenarios.

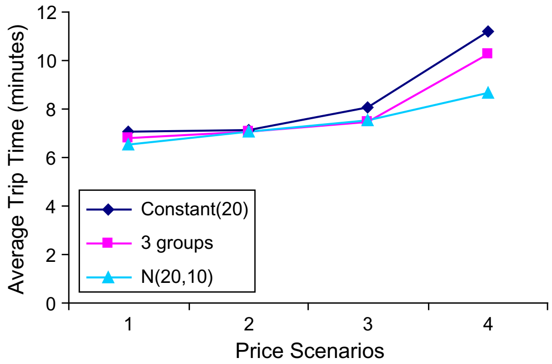

Fig. 8. Average network trip times under different pricing scenarios.

constant VOT and discrete VOT models overestimate the toll road usage when the toll charge is low and underestimate the toll road usage when the toll charge is high.

The impact of underestimation (or overestimation) of the toll road usage under high (or low) toll charge is reflected in the overall network performance. Fig. 8 presents the network-wide average trip times under different pricing scenarios. When the toll charge is high, since the number of trips assigned to the toll road (freeway) is underestimated in the constant VOT and discrete VOT cases and more trips take the low-speed local streets, the resulting average trip time is higher than that of the BDUE model with continuous VOT distribution. Thus, the estimate/prediction of network performance, under a given pricing scenario, obtained from the constant VOT model or discrete VOT model could be biased if user heterogeneity is not captured.

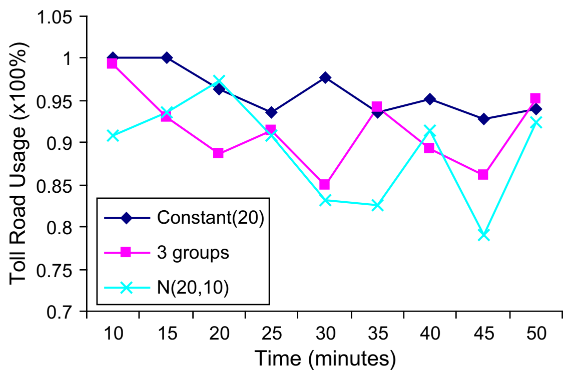

Fig. 9. Time-varying toll road usages under pricing scenario 1.

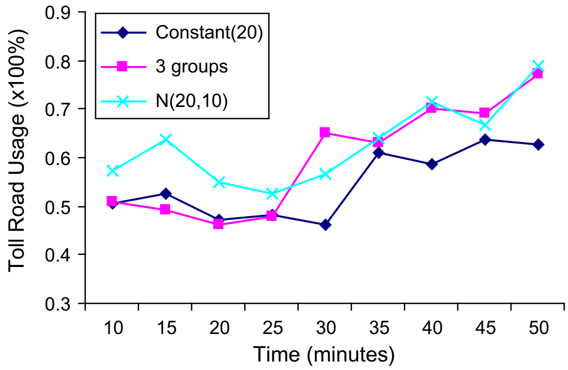

Fig. 10. Time-varying toll road usages under pricing scenario 4.

Figs. 9 and 10 show the time-varying toll road usage of that major OD pair under pricing scenarios 1 and 4, respectively. The two plots further support the findings from Fig. 7, illustrating that, the estimated dynamic toll road utilization under the constant VOT model is higher under low toll charges (i.e. scenario 1), while it is lower under high toll charges (i.e. scenario 4), than that in the discrete and continuous VOT models. It is also seen that when the toll charge increases from period 1 to period 2, the change of toll road usage in the continuous and discrete VOT models is less significant than that in the constant VOT model. This finding provides toll operators with the useful information that when the toll level changes, user reactions may not be as dramatic as predicted by a DUE model with the constant VOT assumption.

6. Concluding remarks

With increasing interest in applying dynamic road pricing strategies to alleviate peak period congestion and improve network performance, there is a need to develop an equilibrium network assignment model capable of capturing traffic dynamics and heterogeneous users' responses to toll charges for the design and evaluation of time-dependent pricing schemes. This paper proposes a bi-criterion dynamic user equilibrium (BDUE) traffic assignment model and presents its solution algorithm. By assuming the value of time (VOT) as a continuously distributed random variable across the population of trips, the BDUE problem is formulated as an infinite dimensional VI. Although the mathematical abstraction of the problem is a typical analytical formulation, this study adopts the simulation-based DTA approach to tackle many practical aspects of the DTA applications. Rather than solving the VI formulation directly, a bi-criterion time-dependent least cost path algorithm, embedded in the simulation-based algorithmic framework, is applied to generate the extreme efficient path set and the corresponding breakpoints that naturally define the multiple user classes, thereby generating the descent direction from a multi-class auxiliary path flow vector.

The experimental results show that the algorithm is able to find close-to-BDUE solutions, and the convergence pattern of the algorithm is similar to that of the classical Frank–Wolfe algorithm and not affected by the different VOT distributions. Using a normal distribution as a benchmark, the constant VOT model overestimates the toll road usage when the toll charge is low and underestimates the toll road usage when the toll charge is high. The impact of estimation biases in terms of the toll road usage is also reflected in the overall network performance, in terms of average trip time. The time-varying toll road usage further highlights that, with the assumption of constant VOT, more trips are assigned to the toll road when the toll charge is low, while fewer trips take the toll road when the toll charge is high. Furthermore, the experiments conducted on the two large road networks (i.e. Irvine and Baltimore–Washington, DC corridor networks) demonstrates the capability of the proposed BDUE algorithm for evaluating large-scale dynamic road pricing scenarios and the advantage of using the vehicle-based implementation technique.

A wide variety of interesting research directions can be continued based on the rich modeling capabilities of the BDUE model in capturing traffic dynamics and user heterogeneity. For example, the BDUE model can be incorporated into a solution framework aiming at finding optimal dynamic pricing schemes, including locations, pricing periods and toll charges, so as to alleviate congestion. The BDUE model can also be applied in designing and evaluating state-dependent high occupancy tolls (HOT) in real-time demand management systems. In addition to the above applications of the BDUE model in optimal pricing problems, an important future research would be to incorporate a more general stochastic path choice approach (e.g. logit, probit or mixed variants) which assumes that users have inaccurate estimates, due to perception errors, of travel times and costs. However, the motivation for such an approach would diminish when real-time information is supplied to users.

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